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Metaphysical Foundations of Natural Science

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IMMANUEL KANT
Metaphysical Foundations of Natural Science

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IMMANUEL KANT

*Metaphysical
Foundations of
Natural Science*

TRANSLATED AND EDITED BY

MICHAEL FRIEDMAN

Stanford University



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Introduction

Much of Kant's intellectual energy, throughout his long career, was devoted to issues in the philosophy of natural science. Kant was not a "philosopher of science" in the sense now familiar within the Anglo-American tradition – a specialist focused on the nature and methods of scientific inquiry, say, or on the foundations of some particular science, such as physics or biology. Kant was a generalist philosopher in the classical sense, concerned with all human thought as such (both practical and theoretical) and with the structure and character of all distinctively human activities and institutions (science, art, religion, law, morality, politics, and so on). Natural science, however, was a particularly central and important example of human thought. Indeed, for the eighteenth century as a whole, the age of Enlightenment and the triumph of Newtonianism, the recent culmination of the scientific revolution of the sixteenth and seventeenth centuries in the work of Newton had elevated natural science to previously undreamt of heights within the intellectual firmament. Thinkers as diverse as Voltaire, Hume, and Kant himself all took the Newtonian achievement in natural science as a model of the human intellect at its best, and as a model, more specifically, for their own philosophical activity.¹

In the eighteenth century, in fact, philosophy as a discipline had not yet clearly split off from natural science, as is indicated by the circumstance that what is now called "natural science" was still often called "natural philosophy" at the time. Moreover, a great stage-setting debate within natural philosophy – the famous correspondence between Leibniz and

¹ See E. Cassirer, *The Philosophy of the Enlightenment* (Princeton: Princeton University Press, 1951); P. Gay, *The Enlightenment: An Interpretation*, 2 vols. (New York: Norton, 1977).

Clarke of 1715–17 – had paid equal attention to both technical problems in physics and natural science (such as the laws of impact and the nature of matter) and very general issues within metaphysics and even theology (such as the principle of sufficient reason and God’s choice to create our world).² In mid-eighteenth-century Germany, in particular, the debate between Leibnizeans and Newtonians dominated the intellectual agenda within both natural science and metaphysics, and Kant himself was no exception. Indeed, his earliest writings were overwhelmingly concerned with problems of natural philosophy in general and the project of reconciling Leibniz and Newton in particular.³

Kant’s early writings in natural philosophy

Two of Kant’s most important “precritical” writings in this connection are the *Universal Natural History and Theory of the Heavens* of 1755 and the *Physical Monadology* of 1756. In the first work Kant developed one of the earliest versions of the “nebular hypothesis.” He formulated the idea that the band of stars visible as the Milky Way consists of a rotating galaxy containing our solar system and that other visible clusters of stars also consist of such galaxies. Moreover, according to the hypothesis in question, all such galaxies originally arose from rotating clouds of gas or nebulae whose centrifugal force of rotation caused a gradual flattening out in a plane perpendicular to the axis of rotation as they cooled and formed individual stars and planets. The laws of such galaxy formation, for Kant, proceed entirely in accordance with “Newtonian principles.” At the same time, however, since our solar system has the same nebular origin as all other galactic structures, we are able to explain one important feature of this system for which the Newtonians had invoked direct

² See H. Alexander, ed., *The Leibniz–Clarke Correspondence* (Manchester: Manchester University Press, 1956). A classical discussion of the underlying issues is A. Koyré, *From the Closed World to the Infinite Universe* (Baltimore: Johns Hopkins University Press, 1957).

³ For an introduction to the situation in German philosophy before Kant see L. Beck, *Early German Philosophy* (Cambridge, Mass.: Harvard University Press, 1969). For Kant’s own intellectual development see E. Cassirer, *Kant’s Life and Thought* (New Haven: Yale University Press, 1981); and, for a more detailed recent treatment, M. Kuehn, *Kant* (Cambridge: Cambridge University Press, 2001). For Kant’s early work in metaphysics and natural philosophy see A. Laywine, *Kant’s Early Metaphysics and the Origins of the Critical Philosophy* (Atascadero, Calif.: Ridgeview, 1993); M. Schönfeld, *The Philosophy of the Young Kant* (Oxford: Oxford University Press, 2000); and the introduction to my *Kant and the Exact Sciences* (Cambridge, Mass.: Harvard University Press, 1992).

divine intervention – the fact that all the planets in our system orbit in approximately the same plane – from purely mechanical natural laws after all, precisely as the Leibnizeans had maintained.⁴

The question dominating the *Physical Monadology* concerned a specific metaphysical problem arising in the debate between the Leibnizeans and the Newtonians. If the ultimate constituents of matter are absolutely simple elementary substances or monads, as the Leibnizeans contended, how can this be reconciled with the geometrical infinite divisibility of space? It would appear that by dividing the space filled or occupied by any given piece of matter, however small, we would also eventually divide the elementary material substances found there as well – contrary to the assumed absolute simplicity of such substances. So how can an elementary constituent of matter or “physical monad” possibly fill the space it occupies without being infinitely divisible in turn? Kant’s answer, in 1756, is that physical monads do not fill the space they occupy by being immediately present in all parts of this space; they are not to be conceived, for example, as bodies that are solid through and through. Physical monads are rather to be conceived as pointlike centers of attractive and repulsive forces, where the repulsive force, in particular, generates a region of solidity or impenetrability in the form of a tiny “sphere of activity” emanating from a central point. Geometrically dividing this region of impenetrability in no way divides the actual substance of the monad, but merely the “sphere of activity” in which the pointlike central source manifests its repulsive capacity to exclude other monads from the region in question. So the Leibnizean commitment to ultimate simple substances or monads is perfectly consistent with the infinite divisibility of space after all – but (and here is Kant’s characteristic twist) it can only be maintained by explicitly adopting the Newtonian conception of forces acting at a distance (in this case a short-range repulsive force acting at a very small distance given by the radius of its “sphere of activity”).⁵

⁴ For a translation of part of Kant’s work see the volume edited by M. Munitz, *Universal Natural History and Theory of the Heavens* (Ann Arbor: University of Michigan Press, 1969). As Kant explains in the preface, he was inspired by ideas of the English astronomers Bradley and Wright. Kant’s contemporary (later friend and correspondent) Lambert published similar ideas, independently of Kant, in his *Cosmological Letters* of 1761. The nebular hypothesis was given its most developed formulation in the eighteenth century by Laplace in his *Système du monde* in 1796. It is now often known as the Kant–Laplace hypothesis.

⁵ See the translation in D. Walford, ed., *Immanuel Kant: Theoretical Philosophy, 1755–1770* (Cambridge: Cambridge University Press, 1992). It is noteworthy that this same solution to the

Kant's conception in the *Physical Monadology* is an early example of a "dynamical theory of matter," according to which the basic properties of solidity and impenetrability are not taken as primitive and self-explanatory, but are rather viewed as derived from an interplay of forces – here, more specifically, the two fundamental forces of attraction and repulsion, which together determine a limit or boundary beyond which repulsion (and thus impenetrability) is no longer effective and attraction (representing Newtonian gravitation) then takes over unhindered. This kind of theory exerted a powerful influence in the later part of the eighteenth century, in the work of such thinkers as Boscovich and Priestley, for example, and it can appropriately be viewed as an anticipation, of sorts, of the field-theoretic approach to physics developed in the nineteenth century, beginning with the work of Faraday and culminating in Maxwell's theory of electricity and magnetism. In this sense, Kant's own contributions to a dynamical theory of matter had a significant impact on the development of natural science itself, quite apart from the original more metaphysical setting within which it was first articulated.⁶

The context of the *Metaphysical Foundations*

The *Metaphysical Foundations of Natural Science* appeared in 1786, at the height of the most creative decade of Kant's "critical" period: the decade of the first edition of the *Critique of Pure Reason* (1781), the *Prolegomena to any Future Metaphysics* (1783), the *Groundwork of the Metaphysics of Morals* (1785), the second edition of the *Critique of Pure Reason* (1787), the *Critique of Practical Reason* (1788), and finally the *Critique of Judgement* (1790). The appearance of this work in 1786 shows, more specifically, that the deep (and in part extraordinarily innovative) concerns with fundamental questions in the natural science and natural philosophy of the time characteristic

problem of absolute simplicity of substance versus geometrical infinite divisibility of space is found in the *Inquiry Concerning the Distinctness of the Principles of Natural Theology and Morality* of 1764, where it appears as an "example of the only certain method for metaphysics illustrated by reference to our cognition of the nature of bodies": see Walford; ed., *Kant*, pp. 259–63.

⁶ For a discussion of the development and influence of eighteenth-century dynamical theories of matter see P. Harman, *Metaphysics and Natural Philosophy* (Brighton, Sussex: Harvester Press, 1982) and *Energy, Force, and Matter* (Cambridge: Cambridge University Press, 1982), as well as E. McMullin, *Newton on Matter and Activity* (Notre Dame: University of Notre Dame Press, 1978), chapter 5. Boscovich's *Theory of Natural Philosophy*, appearing in 1758, was much more widely influential than Kant's *Physical Monadology* – where it again appears that the work of Boscovich and Kant were entirely independent of one another.

of Kant's precritical period were also very salient in the critical period. In particular, the *Metaphysical Foundations* continues, and also attempts to integrate, two separate lines of thought from the precritical period: the extension of Newtonian gravitational astronomy to cosmology first suggested in the *Theory of the Heavens*, and the further development of a dynamical theory of matter as first sketched in the *Physical Monadology*. At the same time, however, Kant now frames both developments within the radically new context of his critical philosophy.

The critical version of the dynamical theory of matter is developed in the longest and most complicated part of the *Metaphysical Foundations*, the second chapter or Dynamics. As in the *Physical Monadology*, Kant here views the basic properties of matter – impenetrability, solidity, hardness, density, and so on – as arising from an interplay of the two fundamental forces of attraction and repulsion. In sharp contrast to the *Physical Monadology*, however, Kant abandons the idea of smallest elementary parts of matter or physical monads and argues instead that all parts of matter or material substances, just like the space they occupy, must be infinitely divisible. Indeed, in the course of developing this argument, he explicitly rejects the very theory of physical monads he had himself earlier defended in 1756. A space filled with matter or material substance, in Kant's new theory, now consists of an infinity or continuum of material points, each of which exerts the two fundamental forces of attraction and repulsion. The “balancing” of the two fundamental forces that had earlier determined a tiny (but finite) *volume* representing a “sphere of activity” of impenetrability around a single pointlike central source now determines a definite *density* of matter at each point in the space in question effected by the mutual interaction of attraction and repulsion.

Thus, in the *Metaphysical Foundations*, as in the *Critique of Pure Reason*, material or phenomenal substance is no longer viewed as simple and indivisible, but is instead a genuine continuum occupying all the (geometrical) points of the space it fills. Accordingly, the problem posed by the infinite divisibility of space that the *Physical Monadology* had attempted to solve by invoking finite “spheres of activity” is now solved, in the Dynamics of the *Metaphysical Foundations*, by invoking the transcendental idealism articulated in the Antinomy of Pure Reason of the first *Critique* – and, more specifically, the argument of the Second Antinomy resolving the apparent incompatibility between the infinite divisibility of space, on the one

side, and the presumed absolute simplicity of the material or phenomenal substances found in space, on the other. Matter or material substance is infinitely *divisible* but never, in experience, ever infinitely *divided*; hence, since matter is a mere appearance or phenomenon and is thus given only in the “progress of experience,” it consists neither in ultimate simple elements nor in an actual or completed infinity of ever smaller spatial parts. Therefore, it is only by viewing matter as a thing in itself or as noumenal substance (which would be necessarily simple) that we obtain a genuine contradiction or antinomy; and so, by an indirect proof or *reductio ad absurdum*, we have a further argument in support of Kant’s characteristically critical doctrine of transcendental idealism.

The cosmological conception presented in the *Theory of the Heavens* had also included a striking vision of how the various galactic structures are distributed throughout the universe. The smallest such structure (due to nebular formation) is our own solar system, consisting of the sun surrounded by the six then known planets. The next larger structure is the Milky Way galaxy, in which our solar system as a whole orbits around a larger center together with a host of other stars and (possible) planetary systems. But the Milky Way galaxy itself, for Kant, is then part of an even larger rotating system consisting of a number of such galaxies; this system is part of a still larger rotating system; and so on *ad infinitum*. The universe as a whole therefore consists of an indefinitely extended sequence of ever larger rotating galactic structures, working its way out from our solar system orbiting around its central sun, through the Milky Way galaxy in which our solar system is itself orbiting around a galactic center, then through a rotating system of such galaxies, and so on. Moreover, this indefinitely extended sequence of galactic structures reflects a parallel indefinitely extended sequence of nebular galactic formation, as the structures in question precipitate out from an initial uniform distribution of gaseous material sequentially starting from the center.

The *Metaphysical Foundations*, unlike the *Theory of the Heavens*, is not a work of cosmology. But the cosmological vision of the *Theory of the Heavens* is still centrally present there, transposed, as it were, into a more epistemological key. The very first explication of the *Metaphysical Foundations*, in the first chapter or Phoronomy, defines matter as the *movable* in space; and, as Kant immediately points out, this inevitably raises the difficult question of relative versus absolute motion, relative versus absolute

space. Kant firmly rejects the Newtonian conception of absolute space as an actual “object of experience,” and he suggests, instead, that it can be conceived along the lines of what he himself calls an “idea of reason.” In this sense, “absolute space” signifies nothing but an indefinitely extended sequence of ever larger “relative spaces,” such that any given relative space in the sequence, viewed initially as at rest, can be then viewed as moving with respect to a still larger relative space found later in the sequence. In the final chapter or Phenomenology, which concerns the question of how matter, as movable, is possible as an object of experience, Kant returns to this theme and develops it more concretely. He characterizes absolute space explicitly as an “idea of reason” and, in this context, describes a procedure for “reducing all motion and rest to absolute space.” This procedure then generates a determinate distinction between true and merely apparent motion – despite the acknowledged relativity of all motion as such to some given empirically specified relative space. The procedure begins by considering our position on the earth, indicates how the earth’s state of true rotation can nonetheless be empirically determined, and concludes by considering the cosmos as a whole, together with the “common center of gravity of all matter,” as the ultimate relative space for correctly determining all true motion and rest.

What Kant appears to be envisioning, then, is an epistemological translation of the cosmological conception of the *Theory of the Heavens*. In order to determine the true motions in the material, and thus empirically accessible universe, we begin with our parochial perspective here on earth, quickly move to the point of view of our solar system (where the earth is now seen to be really in a state of motion), then move to the perspective of the Milky Way galaxy (where the solar system, in turn, is itself seen to be in motion), and so on *ad infinitum* through an ever widening sequence of ever larger galactic structures serving as ever more expansive relative spaces. What Kant calls the “common center of gravity of all matter,” relative to which all the motions in the cosmos as a whole can now be determinately considered, is never actually reached in this sequence; it is rather to be viewed as a forever unattainable regulative idea of reason towards which our sequence of (always empirically accessible) relative spaces is converging. In this way, in particular, we obtain an empirically meaningful surrogate for Newtonian absolute space using precisely the methods used by Newton himself (in determining the true motions in the solar system in the *Principia*, for example). At the same

time, we preserve the fundamental Leibnizean insight that any position in space, and therefore all motion and rest, must ultimately be determined, in experience, from empirically accessible spatiotemporal relations between bodies.⁷

Kant's conception of absolute space in the *Metaphysical Foundations* therefore corresponds – in the more specific context of a consideration of matter as the *movable* in space – to his famous attempt in the *Critique of Pure Reason* to depict his own doctrine of the transcendental ideality of space as the only possible middle ground between the two untenable extreme positions of Newtonian “absolutism” and Leibnizean “relationism.” It also corresponds, even more directly, to Kant's conception of the extent of the material or empirical world in space articulated in the First Antinomy, according to which there is indeed no limit to this extent at any particular finite boundary, but, at the same time, the world cannot be conceived as an actually infinite completed totality nonetheless. In the end, there is only the purely regulative requirement or demand that, in the “progress of experience,” we must always *seek* for further matter beyond any given finite limit and, accordingly, accept no given such boundary as definitive. We must seek, in the terminology of the *Metaphysical Foundations*, for ever larger relative spaces encompassing any given relative space; and, in this way, Kant's conception of absolute space as an idea of reason is the complement, from the point of view of the critical doctrine of transcendental idealism, of his new version of the dynamical theory of matter as consisting of a potential (but not actual) infinity of ever smaller spatial parts. Both are thus now firmly embedded, as we have said, within the radically new critical perspective of “transcendental philosophy.”

Structure of the work: motion and the dynamical theory of matter

Even more obviously, however, there is a quite explicit correspondence between the first *Critique* and the *Metaphysical Foundations* in the very structure of the latter work. It consists of four main parts or chapters which, as Kant explains in the Preface, are coordinated, respectively, with

⁷ For further discussion (in connection, specifically, with Newton's argument for determining the true motions in the solar system in Book III of the *Principia*) see my contribution to P. Guyer, ed., *The Cambridge Companion to Kant* (Cambridge: Cambridge University Press, 1992); and (for even more details) my *Kant and the Exact Sciences* (note 3 above).

the four main headings of the table of pure concepts of the understanding – the categories of quantity, quality, relation, and modality:

The concept of matter had therefore to be carried through all four of the indicated functions of the concepts of the understanding (in four chapters), where in each a new determination of this concept was added . . . The *first* considers *motion* as a pure *quantum* in accordance with its composition, without any quality of the movable, and may be called **phoronomy**. The *second* takes into consideration motion as belonging to the *quality* of matter, under the name of an original moving force, and is therefore called **dynamics**. The *third* considers matter with this quality as in *relation* to another through its own inherent motion, and therefore appears under the name of **mechanics**. The *fourth* chapter, however, determines matter's motion or rest merely in relation to the mode of representation or *modality*, and thus as appearance of the outer senses, and is called **phenomenology**. (Ak 4:476–77)

Hence, the *Metaphysical Foundations* is explicitly constructed and organized by the guiding “architectonic” of the critical period – the structure first given by what Kant calls the table of logical functions of a possible judgment.⁸

Thus, the first chapter or Phoronomy subsumes the concept of matter as the movable in space under the categories of quantity by showing how it is possible to consider such motion as a mathematical magnitude – to show, more specifically, how the concept of speed or velocity first acquires a mathematical structure. This, according to the general concept of quantity or magnitude considered in the first *Critique*, requires that we show how any two magnitudes falling under a common magnitude kind (two lengths, areas, or volumes, for example) may be composed or added together so that a new magnitude having the properties of the mathematical sum of the two then results. Our problem, in the present case, is therefore to show how any two speeds or velocities may be summed or

⁸ The particularly close connection between the *Metaphysical Foundations* and the first *Critique* in this regard is further emphasized in the second (1787) edition of the *Critique*, where Kant remarks, as a comment to the table of categories, that (B109–10) “[this] table contains all elementary concepts of the understanding completely, and even the form of a system of such concepts in the human understanding; and it therefore gives an indication of all the *moments* of a prospective speculative science, and even their *ordering*, as I have also attempted to show elsewhere* [**Metaphysical Foundations of Natural Science*].”

added together; and, since motion must always be defined with respect to one or another “relative space” (what we would now call a reference frame), it is also necessary carefully to consider the (possibly different) such spaces involved. Kant’s main contention, in the single proposition of the Phoronomy, is that the addition or composition in question cannot be carried out in a single relative space or reference frame. Rather, we must consider two different spaces or reference frames, such that the moving body has the first velocity with respect to one of the spaces, while this space, in turn, moves with the second velocity with respect to a second relative space: the motion of the body with respect to this second space or reference frame then has the desired sum of the two original velocities. Moreover, this whole “construction,” for Kant, rests on a principle of the relativity of motion (according to which the two different reference frames in question are in an important sense equivalent); and it thus depends essentially on Kant’s characteristic conception of the relativity of space and of motion already discussed above in connection with the consideration of absolute versus relative space at the very beginning of the Phoronomy.

In the Phoronomy, as Kant explains, the moving “body” or piece of “matter” in question is considered as having only the properties of speed and direction, and is thus entirely bereft of all empirically given “qualities” – mass, density, force, and so on – possessed by real physical bodies or bits of matter. Indeed, the “bodies” of the Phoronomy, as Kant explicitly points out, can, in this respect, be considered as mere mathematical points. The role of the second chapter or Dynamics, therefore, is precisely to explain how such empirical qualitative features are first introduced: how it is possible, in the words of the first explication, that matter, as the movable, then *fills* a space. The answer to this question turns out to be long and complicated, and to invoke the full resources, as we have already suggested, of Kant’s critical version of the dynamical theory of matter. Matter fills the space it occupies by a continuous “balancing” of the two fundamental forces of attraction and repulsion exerted by all the continuum of points in the space in question. Repulsive force, however, has priority here; for, as Kant also says in the first explication, to fill a space, in the first instance, means to resist penetration (by other matter) into this space. Accordingly, the main body of the Dynamics chapter is organized into two symmetrical parts, where the first

four propositions and accompanying remarks discuss the fundamental force of repulsion and the last four focus on the fundamental force of attraction.

Kant argues, in the first four propositions, that the impenetrability effected by repulsive force must be conceived as relative rather than as absolute. Matter is not a perfectly hard or impenetrable solid that resists penetration or compression absolutely; it is rather an essentially elastic continuum exerting expansive force or pressure against any attempt to compress it into a smaller space. The more it is compressed the more it resists, but there is no matter that cannot be compressed at all. Matter is thus what Kant calls an originally elastic or expansive medium (a concept he illustrates by the air filling the barrel of an air pump). Moreover, since Kant now defines material substance (such as air or any other “elastic fluid,” for example) as that which is movable independently of any other matter, and since the originally expansive elasticity characteristic of such a substance can only arise, Kant argues, from repulsive forces exerted by *every* point in the space it fills, it follows that material substance is found in *all* of the parts of this same space (for each part exerts repulsive force against its neighboring parts and is thereby physically separable from them). Material substance must now be conceived, therefore, as essentially divisible to infinity, and Kant’s critical version of the dynamical theory, as we have already explained, is thus in explicit opposition here to his earlier precritical theory developed in the *Physical Monadology* – where matter is only finitely divisible into elementary corpuscles or physical monads representing ultimate simple substances.

Kant introduces the fundamental force of attraction, in the next four propositions, by his critical version of the “balancing” argument. If matter had only the fundamental force of repulsion, Kant argues, it would expand itself to infinity by its own internal pressure – and, in this case, matter would have zero density everywhere, and space would turn out to be empty. In order that matter be really possible as that which fills a space, therefore, there must be something that resists this internal pressure. This, Kant concludes, can only be a second fundamental force essentially opposed to the fundamental force of repulsion: namely, a fundamental force of attraction. In order for matter to fill the space it occupies to a determinate degree – to have a determinate density or “quantity of matter” within this space – there must thus be a continuous “balancing,”

at each point of the space in question, of both fundamental forces.⁹ In the remainder of the second part of the Dynamics Kant then explains that the properties of the fundamental force of attraction are precisely those of Newtonian universal gravitation: in particular, it acts immediately at a distance independently of any intervening matter (in Kant's terminology, it is a "penetrating" rather than a "surface" force), and it acts in this way between each part of matter in the universe and any other part at arbitrary distances to infinity. Kant connects this discussion, both implicitly and explicitly, with some of the main steps in Newton's own argument for universal gravitation presented in Book III of the *Principia*.

The eight propositions of the Dynamics describing the properties of the two fundamental forces are, as Kant explains, concerned only with completely "universal" properties of all matter in general and as such – properties or qualities "comprehensible a priori" by his own "metaphysical" treatment. These properties include original elasticity due to the fundamental force of repulsion and weight due to the fundamental force of attraction, together with the closely related properties of density and "quantity of matter." Nothing else, Kant suggests, can be comprehended a priori, and so all other properties of matter than these belong to a "physical" rather than properly "metaphysical" discussion. Nevertheless, in the long General Remark that concludes the Dynamics, Kant indicates, in a more or less speculative spirit, how some of the main headings of a physical treatment of the particular properties of matter responsible for its "specific variety" might be set up. He here discusses the distinction between fluid and solid matter, the property of cohesion (or attraction in contact), rigidity, elasticity in the sense of "spring force," and, finally, some of the key concepts of the discipline of chemistry (which, according to the Preface, is not yet a genuine science in the strict sense). Kant concludes this discussion with some important methodological remarks concerning the general approach one should take to all such properly physical questions. In general, the "metaphysical-dynamical" approach, which views matter as a true dynamical continuum and eschews absolutely hard elementary corpuscles and empty space, is to be preferred

⁹ Again, as I pointed out above, this contrasts with Kant's precritical presentation of an analogous "balancing" argument in the *Physical Monadology*, according to which the interplay of attraction and repulsion determines a tiny (but) finite *volume* – what Kant calls the "sphere of activity" of impenetrability – around a pointlike central source.

to the opposing “mathematical-mechanical” approach, which postulates precisely an interspersing of hard elementary corpuscles and empty space from the very beginning (to explain differences of density) – and, in its extreme form, thereby attempts to dispense with all genuinely dynamical forces originally inherent in matter.

The third chapter or Mechanics considers matter (the movable in space) insofar as it has a “moving force” *as* a movable thing. Whereas the moving forces considered in the Dynamics – the fundamental forces of attraction and repulsion – are present or inherent in the bodies exerting them entirely independently of the state of motion of these bodies, now, in the Mechanics, we consider these same moving forces as involved in what Kant calls the communication of motion from one body to another. A paradigmatic instance of such communication of motion is impact, whereby one body transfers motion to another in virtue of their forces of impenetrability and loses as much motion through the impact as the impacted body gains. Kant makes it clear, however, that the very same phenomenon takes place in cases of attraction – where, for example, the attracting body produces a motion in the attracted body and, at the same time, is also in motion itself due to the (mechanical) resistance of the attracted body. In all such cases of the communication of motion, quite generally, the operative mechanical quantity is momentum (or mass times velocity), and any change of momentum produced by one body on another is precisely counterbalanced by an equal and opposite change of momentum experienced by the first body. *Both* bodies involved must necessarily be viewed as in motion, and the total momentum is necessarily conserved.

The fundamental importance of the concept of momentum here affords Kant an opportunity to explain more precisely the central concept of quantity of matter, which had been introduced into the Dynamics in connection with the density of matter. Quantity of matter is now officially defined, in the second explication of the Mechanics, as the (continuously extended and infinitely divisible) “aggregate of movables” in a given space. In accordance with the discussion in the Dynamics, in particular, the same quantity of matter present in a larger space can be brought into a smaller space by compression, where it is then correspondingly more dense than it was before (and vice versa for expansion). Quantity of matter, as in Newton, can thus be conceived as depending on both volume and density. But this concept, Kant explains, affords us no universally applicable measure of the quantity of matter, for matters of specifically different

kinds (water and mercury, for example) cannot be transformed into one another by compression.¹⁰ The only universally valid way to estimate quantity of matter, according to Kant's first proposition of the *Mechanics*, involves considering the momenta, and thus possible motions, of the bodies in question. As Kant explains, more precisely, quantity of matter manifests itself in experience only by the quantity of motion (i.e., momentum) at a given speed. Thus, for example, when we compare the quantities of matter of two substances in equilibrium on a balance, the two press down on the balance with equal gravitational forces and therefore manifest equal changes of momentum; but since, by Galileo's law of fall, their initial speeds (and thus accelerations) are equal, it follows that their masses or quantities of matter must be equal as well. And, more generally, it is only in the context of the communication of motion – and thus “mechanically” – that mass or quantity of matter can be validly estimated.

Instantiating the categories in space

Kant then moves, in the last three propositions of the *Mechanics*, to the main business of the chapter: establishing what he calls the three “laws of mechanics.” These are, first, a principle of the conservation of the total quantity of matter in the universe, second, a version of the law of inertia, and third, the law of the equality of action and reaction. Here we find a very explicit correspondence, as promised, between the subject matter of the *Mechanics* and the categories of relation. In particular, the principle of the conservation of the total quantity of matter corresponds to the more general “transcendental” principle established in the first *Critique* – the permanence of *substance* in all changes in the (phenomenal) world; the law of inertia corresponds to the category, and accompanying principle, of *causality*; and the law of the equality of action and reaction corresponds to the category, and accompanying principle, of thoroughgoing dynamical interaction or *community*. Thus, in considering material substances or bodies as interacting with one another through their fundamental forces and, as a result, thereby standing in relation to one another in a community

¹⁰ This particular point, in the context of the specific example of water and mercury, is made in Kant's earlier discussion of density in the first number of the General Remark to *Dynamics* (see 4:525–26).

of their inherent motions (i.e., momenta), we are, at the same time, applying the categories or pure concepts of relation to these same bodies.

It is in precisely this context, in fact, that Kant makes his most explicit and developed remarks about the relationship between the very general “transcendental” principles established in the first *Critique* and the more specific “metaphysical” principles established in the *Metaphysical Foundations* – remarks which are especially salient in the second (1787) edition of the first *Critique*. For example, in the second edition Introduction, Kant formulates the question of how pure natural science is possible, and he then adds a footnote instancing the laws of “the permanence of the same quantity of matter, of inertia, [and] of the equality of action and reaction” as clear and uncontroversial examples of such pure natural science (B20). Later, even more strikingly, he adds an entirely new section to the chapter on the system of principles of pure understanding, a “General Remark to the System of Principles.” Here Kant first argues that the pure categories, without corresponding (spatiotemporal) intuitions, remain mere empty forms of thought, and he then argues for the “even more remarkable” conclusion that the categories require “not merely intuitions, but always even *outer intuitions*” (see B291–94). In order to have a permanent intuition corresponding to the category of substance, for example, we require “an intuition *in space* (of matter).” In order to instantiate the category of causality we require an intuition of change or alteration, and this intuition can only be “that of the motion of a point in space, whose existence in different places (as a sequence of opposed determinations) first makes alteration intuitive for us.” Finally, we can similarly make the possibility of community comprehensible to ourselves only when we “represent it in space, and thus in outer intuition” – “for the latter already contains in itself *a priori* formal outer relations as conditions for the possibility of real relations (in action and reaction, and therefore community).”¹¹

¹¹ Further changes in the second edition of the first *Critique* also clearly reflect the importance (and influence) of the intervening *Metaphysical Foundations*. Thus, for example, the second edition reformulates the First Analogy so that it now expresses a *conservation law* for the total quantity of substance (B224): “In all change of the appearances substance is permanent, and its quantum in nature is neither increased nor diminished.” And there are parallel, if less dramatic changes made to the other two Analogies. Moreover, the Refutation of Idealism added to the second edition explains that “we have nothing permanent that could underlie the concept of a substance, as intuition, except merely *matter*” (B278), and thus it mirrors the above cited General Remark (which, in turn, explicitly refers to the Refutation of Idealism). Compare also note 8 above.

This last passage added to the second edition of the *Critique* closely mirrors a corresponding passage in the Preface to the *Metaphysical Foundations* itself:

It is also indeed very remarkable (but cannot be expounded in detail here) that general metaphysics, in all instances where it requires examples (intuitions) in order to provide meaning for its pure concepts of the understanding, must always take them from the general doctrine of body, and thus from the form and principles of outer intuition; and, if these are not exhibited completely, it gropes uncertainly and unsteadily among mere meaningless concepts . . . And so a separated metaphysics of corporeal nature does excellent and indispensable service for *general* metaphysics, in that the former furnishes examples (instances *in concreto*) in which to realize the concepts and propositions of the latter (properly speaking, transcendental philosophy), that is, to give a mere form of thought sense and meaning. (4:478)

Kant had already explained in the Preface that the special metaphysics of corporeal nature expounded in the *Metaphysical Foundations* differs from the general metaphysics articulated in the first *Critique* by limiting itself to the objects of specifically outer, that is spatial, intuition. It would appear, however, that there is nonetheless an especially close connection between the more general concepts and principles of the first *Critique* and the more specific concepts and principles of the *Metaphysical Foundations*. Indeed, Kant here suggests that the *only* way we can realize or instantiate concretely the abstract concepts and principles of transcendental philosophy is precisely by the objects of specifically outer intuition – by matter as the movable in space.

True and apparent motion

The fourth chapter or Phenomenology of the *Metaphysical Foundations* considers how matter as the movable in space can be an object of experience with regard to its state of motion – how, as Kant puts it, matter as the movable in space can be thought as *determined*, one way or another, by the predicate of motion. The underlying problem, as Kant makes clear, concerns the relativity of space and of motion first broached in the Phoronomy. In the Phoronomy, however, a principle of the thorough-going relativity of all motion appeared to hold unlimited sway, in that

it was considered as all the same whether a body is viewed as being in motion (with respect to some empirically defined relative space or reference frame) or at rest (with respect to another such space or reference frame). Now, in the *Phenomenology*, the problem is precisely to explain how a body can be definitely and unequivocally characterized as being in one particular state to the exclusion of the other – how, in other words, we can apply a definite and unequivocal distinction between true and merely apparent motion. The main body of the *Phenomenology* consists of three propositions and accompanying remarks, which correspond, respectively, to the three categories of modality: possibility, actuality, and necessity. Thus, the rectilinear motion of a body with respect to a given (empirical) space, as distinct from the opposite motion of the space itself, is a merely *possible* (merely relative) predicate; the circular motion of a body, as distinct from the opposite motion of the (surrounding) space, is an *actual* (or true) predicate; and the mutual relative motions of two bodies, if either one is first assumed to be in motion relative to the other, is a *necessary* characterization of both bodies.

When Kant first articulates his principle of the relativity of motion in the *Phoronomy*, he already explains that it is subject to two important qualifications. In the first place, he says, he is assuming all the relevant motions to be rectilinear; for, “in regard to curvilinear motions, it is not in all respects the same whether I am authorized to view the body (the earth in its daily rotation, for example) as moved and the surrounding space (the starry heavens) to be at rest, or the latter as moved and the former as at rest, which will be specifically treated in what follows” (4:488; the “following” treatment of this case turns out to be the second proposition of the *Phenomenology*). In the second place, Kant explains, even the rectilinear motion of one body relative to another is not as completely arbitrary as it first appears. Whereas, “in phoronomy, where I consider the motion of a body only in relation to the space (which has no influence at all on the rest or motion of the body), it is completely undetermined and arbitrary how much speed, if any, I wish to ascribe to the one or the other,” it later turns out, “in mechanics, where a moving body is to be considered in active relation to other bodies in the space of its motion, this will no longer be entirely the same, as will be shown in the proper place” (*ibid.*; thus the “proper place” in question turns out to be in the *Mechanics*). These two qualifications already introduced in the *Phoronomy* then correspond to the second and third propositions of

the Phenomenology, where the second determines the actuality of circular (and more generally curvilinear) motion, and the third determines the necessity of equal and opposite motions of both bodies in accordance with the fourth proposition (the “Third Law of Mechanics”) established in the Mechanics.¹²

The fourth proposition of the Mechanics, as we have already seen, formulates the equality of action and reaction: whenever one body acts on another by either the fundamental force of repulsion (as in cases of impact) or the fundamental force of attraction (as in universal gravitation), an equal and opposite reaction – that is, an equal and opposite motion or change of momentum – is experienced by the first body. *Both* bodies must necessarily be viewed as moving, and, as Kant emphasizes repeatedly, no motion at all can be communicated to a body absolutely at rest. Kant proves the proposition by showing how any motions arising in this way are to be “reduced to absolute space.” In cases of impact, for example, there is a privileged frame of reference determined by the *center of mass* of the two bodies, such that both bodies are moving towards one another before the impact with equal and opposite momenta. The speed of the first body is to that of the second as the mass of the second body is to that of the first; and it is precisely this particular way of apportioning speeds between the two bodies that resolves the arbitrariness left open in cases of rectilinear motion by the Phoronomy. The true as opposed to merely apparent rectilinear motions, then, are precisely those involving equal and opposite motions (i.e., momenta) of both bodies involved in the communication of motion. The center of mass of the two bodies provides an empirically accessible surrogate for absolute space, and the motions determined with respect to any other frame of reference (relative to which one of the two bodies is initially at rest, for example) are merely relative or apparent. Moreover, as Kant also suggests in the Mechanics, the situation is quite similar in cases of attraction: here, too, the center of mass of the two interacting bodies, relative to which both bodies are necessarily in motion, provides us with a privileged frame of reference for describing the motions in question.

¹² More generally, Kant explains, the first proposition of the Phenomenology determines “the modality of motion” with respect to phoronomy, the second with respect to dynamics, and the third with respect to mechanics.

The case of attraction turns out to be rather more complicated, however. For the motions typically effected by attraction, in the guise of universal gravitation, are rotational or orbital motions rather than rectilinear motions. Here the two interacting bodies both orbit around a common central point, their mutual center of mass or center of gravity, but, unlike in the case of impact, they do not necessarily move along the straight line between them relative to this central point. Determining the center of mass lying on the straight line between them does not yet suffice, in any case, to determine the state of true or actual *rotational* motion – where the issue concerns whether this line itself rotates around its central point.¹³ The solution to this problem is presented in Kant’s second proposition of the *Phenomenology*, where he refers, in particular, to Newton’s remarks in the Scholium to the Definitions of the *Principia* showing how the circular motion of two globes connected by a cord around a common center can be empirically determined from the resulting tension in the cord due to centrifugal force. And it is clear, from the context, that Newton intends this illustration as a model for his later argument in Book III of the *Principia* showing how the true (rotational and orbital) motions in the solar system can be determined via his theory of gravitation. Here the force of gravity takes the place of the connecting cord, and the various orbital motions around different common centers (the earth and its moon around their common center of gravity, the earth and the sun around their common center of gravity, and so on) are given a single unified representation by taking the common center of gravity of the sun and all the planets as determining the privileged frame of reference for considering all of these motions together.

Absolute space as an idea of reason

The General Remark to *Phenomenology* sketches Kant’s procedure for “reducing all motion and rest to absolute space.” The discussion suggests

¹³ In the case of the orbital motions of two bodies around a common central point they do not move (rectilinearly) either towards or away from this point if the orbits are circular. If they are elliptical, however, then the resulting motion has two components: a rotational motion of the straight line connecting the two bodies around the central point plus a (rectilinear) oscillating motion of the bodies towards and away from this central point. In the course of its yearly elliptical orbit around the sun, for example, the earth moves closer to the sun (more precisely, to the common center of gravity of the earth–sun system) in one part of its orbit and farther from the sun in another part.

that we begin this procedure from our naïve perspective on the earth, initially taken to be in a state of rest. But all motions viewed from the point of view of the earth at rest are so far merely relative and, in accordance with the first proposition, therefore merely possible (for, once again, no body whatever can be truly or actually in a state of absolute rest). The next step is taken when we determine the state of true axial rotation of the earth (relative to the surrounding starry heavens), and this case, as Kant suggests, is analogous, in important respects, to the Newtonian example of the two (orbitally) rotating bodies connected by a cord. What Kant suggests, more specifically, is that we determine the true or actual axial rotation of the earth by observing how the earth's eastward rotation deflects a falling body from its downward rectilinear path towards the earth's center by what we now call Coriolis force. Here, then, we have a combined effect of gravitational and Coriolis forces, which is precisely parallel, as Kant also suggests, to the more familiar balancing of gravitational and centrifugal forces.¹⁴ In all such cases, more generally, the actually observed motions result from the combination of a centrifugal tendency to proceed in a straight line tangent to the circular (or more generally curvilinear) motion in question, in accordance with the law of inertia, and a centripetal tendency to proceed in a straight line towards the center of circular (or more generally curvilinear) motion, in accordance with the law of gravitation. That gravitational attraction is here counterbalanced by circular (or curvilinear) *motion* – “without any dynamical repulsive cause” – is precisely what indicates the actuality of this motion.¹⁵

¹⁴ It is worth noting, at this point, that two of Kant's most prescient and original contributions to natural science concerned effects arising from a combination of the earth's rotation and gravitational forces. *Whether the Earth has Undergone an Alteration of its Axial Rotation*, appearing in 1754 (and reprinted in the volume edited by Munitz cited in note 4 above), was the first work (by more than one hundred years) to take seriously the possibility that tidal friction arising from the attraction of the sun and the moon on the sea could produce nonuniformities in the earth's rate of rotation. Similarly, the *Theory of the Winds*, appearing in 1756, was the first work (by almost eighty years) to suggest an explanation of the directional tendency of the trade winds by the rotation of the earth – in terms of Coriolis forces due to this rotation, which upset the hydrostatic equilibrium in the atmosphere maintaining equal pressures at equal distances from the earth by a balancing of gravitational force and the air's expansive elasticity.

¹⁵ Kant makes the remark about the absence of any “dynamical repulsive cause,” and explicitly refers again to Newton's Scholium to the Definitions, at 4:562. The point is that the earth's gravitational attraction on the moon, for example, is balanced by the moon's orbital motion rather than any counteracting dynamical repulsive force – if the moon ceased orbiting it would drop in a straight line towards the earth. Similarly, in the case of Newton's example of the two globes, the tension in the cord is produced precisely by their mutual rotation and not by any repulsive force – if the rotation were to cease then so would the tension in the cord.

The remainder of Kant's discussion is unfortunately extremely compressed. He reminds us that mutually opposed and equal motions of any two bodies interacting by any "dynamical influences" whatsoever (he gives as examples "gravity or a tensed cord") are always necessary; he then moves rapidly, as we have already pointed out, to a consideration of "the common center of gravity of all matter." Thus, although Kant does explicitly refer, once again, to Newton's Scholium to the Definitions here (see note 15), he does not refer to the actual argument of Book III. Nevertheless, in light of Kant's earlier implicit and explicit references to the main steps in the argument of Book III in the Dynamics, it is plausible to suppose, as we have also already suggested, that Kant is in fact envisioning a cosmological extension of just this Newtonian argument. After determining the earth's state of true rotation, we then take up the perspective of the center of mass of the solar system, relative to which all rotational and orbital motions within this system can be given a unified representation within a single frame of reference; we proceed from there to the center of mass of the Milky Way galaxy, from there to the center of mass of a system of such galaxies, and so on *ad infinitum*. It is in precisely this way, as pointed out above, that we obtain an empirically meaningful surrogate for Newtonian absolute space, now reconceived as a regulative idea of reason, whereby all true or actual motions in the universe can be eventually effectively determined.¹⁶

Kant concludes the Phenomenology by reflecting on various concepts of *empty space*: phoronomical, dynamical, and mechanical. His main point, with reference to the earlier discussion in the Dynamics, is that there is no need to assume empty spaces interspersed within otherwise perfectly hard or solid matter to explain observed differences in density (and thus differences in quantity of matter). Kant concludes, therefore, by pointing, once again, to the indefinitely extended regress towards smaller and smaller parts of (continuously distributed) matter characteristic of Kant's critical version of the dynamical theory – which regress, as we have seen, is complementary, within his critical system, to the above progress towards larger and larger (rotating) systems of matter converging (in the limit) towards what he calls absolute space.

¹⁶ See the discussion to which note 7 above is appended and, for further details and argument, the references cited there.

Reading the *Metaphysical Foundations*

The *Metaphysical Foundations* is difficult to read, even by Kantian standards. The argument is often very compressed, and the text appears to have been written rather hastily. In addition, the quasimathematical style of presentation (which it shares with the *Physical Monadology* and some other precritical works) – its organization into definitions (explications), propositions, proofs, remarks, and so on – is quite formidable and creates further difficulties for the reader. It is therefore especially important in this case to supply some of the missing context on one's own. It helps a great deal, in particular, to connect the text with both other works in natural science and natural philosophy against the background of which Kant was writing, and with his own precritical writings that he is here in the process of revising and developing.

The author to whom Kant explicitly refers most often – far more often than any other – is Newton, and most such references are to the *Principia*. So it greatly helps to read the *Metaphysical Foundations* with the *Principia* ready to hand, and to track down all references, both explicit and implicit, to this work. Some of the more obvious implicit references are recorded in my notes, but it pays to be alert for other points of comparison as well. When Kant gives an official definition of something like “quantity of matter,” for example, it is illuminating to consult Newton's definition of the same concept. In addition to the *Principia*, Kant also refers, both explicitly and implicitly, to the *Optics*; here again, one should be alert for such references as well, which are typically to the *Queries* at the end of the *Optics*. But Newton is not, of course, the only writer in natural science and natural philosophy to whom Kant refers and with whom he was quite familiar. He explicitly refers, for example, to Descartes, Euler, Kepler, Lambert, Leibniz, and Mariotte. In the case of Leibniz, the correspondence with Clarke was of course particularly salient, along with other Leibnizean texts with which Kant was familiar, such as the *Theodicy*, *New Essays Concerning Human Understanding*, and *Monadology*. In the case of Euler, his *Letters to a German Princess on Different Subjects in Natural Philosophy* (where, for example, he describes his important wave theory of light) exerted a deep influence on both the eighteenth-century discussion of such issues in general and on Kant's own thinking in particular.

Among Kant's precritical writings, as I have already explained, the *Theory of the Heavens* and *Physical Monadology* are especially central

and important pieces of the background to the *Metaphysical Foundations*. Also important, however, are the *New Elucidation of the First Principles of Metaphysical Cognition* (1755), *The Only Possible Argument in Support of a Demonstration of the Existence of God* (1763), the *Attempt to Introduce the Concept of Negative Magnitudes into Philosophy* (1763), the *Inquiry Concerning the Distinctness of the Principles of Natural Theology and Morality* (1764), *Dreams of a Spirit-Seer* (1766), *Concerning the Ultimate Ground of the Differentiation of Directions in Space* (1768), and the *Inaugural Dissertation* (1770).¹⁷ The *New Elucidation* and *Physical Monadology* constitute a particularly important pair of works here, for the *New Elucidation* stands to the *Physical Monadology* as the *Critique of Pure Reason* stands to the *Metaphysical Foundations*: the former work presents the more general metaphysical framework within which the more specific natural philosophical discussions in the latter then proceed. So here it is especially illuminating, for example, to consider how the concept of a “force of inertia,” as discussed in both the *New Elucidation* and the *Physical Monadology*, is intertwined with the more general monadological conception of substance articulated in both works – and to read the explicit rejection of this concept in the *Mechanics* of the *Metaphysical Foundations* against this precritical background.

Some of the most obvious points of connection and correspondence between the *Critique of Pure Reason* and the *Metaphysical Foundations* have already been discussed above. And it is clear that the *Critique of Pure Reason*, more generally, provides the overarching philosophical context for the more specific issues in natural science and natural philosophy with which Kant is occupied in the *Metaphysical Foundations*. One should, therefore, be alert to this context at all times. For example, since, as we have seen, each main chapter of the *Metaphysical Foundations* corresponds to a heading of the principles of pure understanding as discussed in the *Transcendental*

¹⁷ All of these are translated in the volume edited by Walford cited in note 5 above (with the exception of the *Theory of the Heavens*, for which see note 4). For a discussion of the natural philosophical significance of *Dreams of a Spirit-Seer* see the work of Laywine cited in note 3. Kant’s correspondence with Lambert is especially important for Kant’s intellectual development prior to and in connection with the *Inaugural Dissertation*; it is translated in A. Zweig, ed., *Correspondence* (Cambridge: Cambridge University Press, 1999). (In general, Lambert and his influence on Kant are helpfully described in the work of Beck cited in note 3.) Another particularly important precritical work (from the point of view of the *Metaphysical Foundations*), which has unfortunately not yet been translated into English, is the *Neuer Lehrbegriff der Bewegung und Ruhe* (*New System of Motion and Rest*) of 1758.

Analytic – the Axioms of Intuition, Anticipations of Perception, Analogies of Experience, and Postulates of Empirical Thought, respectively – one should always read the corresponding chapters of the *Metaphysical Foundations* in relation to these discussions in the Transcendental Analytic. But of course Kant’s theory of space and time in the Transcendental Aesthetic is also very important, as well as such parts of the Transcendental Dialectic as the Antinomies of Pure Reason and the Appendix on the regulative use of reason. More generally, whenever Kant discusses space, motion, force, or matter in the first *Critique*, such discussions illuminate (and are illuminated by) the corresponding discussions of these concepts in the *Metaphysical Foundations*.

There is a final issue about reading the *Metaphysical Foundations* that concerns the structure and organization of the text itself. Although Kant is, in a sense, presenting a continuous linear argument, it is often the case that earlier arguments point towards later parts of the text for their completion and full articulation. Thus, as we have seen, Kant formulates a principle of the relativity of motion in the Phoronomy, but he also explicitly qualifies it there with respect to issues that are only later discussed in the Mechanics and Phenomenology. Similarly, he gives an argument for the infinite divisibility of material substance, and explicitly opposes the *Physical Monadology* in this regard, in the Dynamics; he returns to this question, and again opposes the *Physical Monadology* on the same issue, in the later discussion of quantity of matter and its conservation in the Mechanics – where, in particular, the concept or category of substance is now more explicitly salient. More generally, then, one should always be alert to the many and varied ways in which Kant anticipates later discussions earlier in the text and, conversely, refers back to earlier parts of the text in later discussions. In this sense, the text is more “dialectical” than linear, in that the meaning and point of what Kant is saying, at any given stage, only becomes fully articulated at a later stage. In the end, there is no alternative to reading and rereading Kant’s text repeatedly, while patiently attempting to assemble all the pieces of the puzzle bit by bit; in my experience, however, it more than amply repays the effort.

Chronology

- 1724 Immanuel Kant born April 22 in Königsberg, East Prussia
- 1730–32 Attended Vorstädter Hospitalschule (elementary school)
- 1732–40 Attended the Collegium Fridericianum (parochial – Pietist – school)
- 1740–46 Attended the University of Königsberg
- 1747 *Thoughts on the True Estimation of Living Forces*
- 1747–54 Served as private tutor for families in the vicinity of Königsberg
- 1755 Completed dissertation entitled “Succinct Exposition of Some Meditations on Fire” and received his doctoral degree from the Faculty of Philosophy at the University of Königsberg
- Universal Natural History and Theory of the Heavens*, in which Kant proposed an astronomical theory now known as the Kant–Laplace hypothesis
- New Elucidation of the First Principles of Metaphysical Cognition*, paper presented to the Philosophy Faculty
- 1756 Three treatises on an earthquake in Lisbon
- Physical Monadology*
- 1762 *The False Subtlety of the Four Syllogistic Figures*
- 1763 *The Only Possible Argument in Support of a Demonstration of the Existence of God*
- 1764 *Observations on the Feeling of the Beautiful and the Sublime*
- 1764 *Inquiry Concerning the Distinctiveness of the Principles of Natural Theology and Morals*
- 1766 *Dreams of a Spirit-Seer elucidated by Dreams of Metaphysics*

- 1770 Appointed Professor of Logic and Metaphysics at the University of Königsberg; inaugural dissertation entitled *Concerning the Form and Principles of the Sensible and the Intelligible World*
- 1781 *Critique of Pure Reason*, first (A) edition
- 1783 *Prolegomena to Any Future Metaphysics*
- 1784 *Ideas Towards a Universal History from a Cosmopolitan Point of View*
An Answer to the Question: What is Enlightenment?
- 1785 Review of Herder's *Ideas for a Philosophy of the History of Mankind*
Groundwork of the Metaphysics of Morals
- 1786 Elected to the Academy of Sciences in Berlin
Conjectural Beginning of Human History
Metaphysical Foundations of Natural Science
What is Orientation in Thinking?
- 1787 *Critique of Pure Reason*, second (B) edition
- 1788 *Critique of Practical Reason*
Concerning the Use of Teleological Principles in Philosophy
- 1790 *Critique of Judgement*, first edition
- 1793 *On the Proverb: That May be True in Theory but is of No Practical Use*
Critique of Judgement, second edition
Religion within the Limits of Reason Alone
- 1794 Censured by the Imperial Censor; elected to the Academy of Sciences, St. Petersburg
The End of All Things
- 1795 *On Perpetual Peace*
- 1796 July: Kant's last lecture
- 1797 *Metaphysics of Morals*
On the Supposed Right to Tell Lies from Benevolent Motives
- 1798 *Anthropology from a Pragmatic Point of View*
The Conflict of the Faculties (Part II: "An Old Question Raised Again: Is the Human Race Constantly Progressing?")
- 1800 *Logic*
- 1803 Kant becomes ill
Education (Pedagogy)
- 1804 Kant dies on February 12th, buried February 28th

Further reading

Serious discussion in the English-speaking world of Kant's philosophy of natural science in general and the *Metaphysical Foundations* in particular begins with Gerd Buchdahl, *Metaphysics and the Philosophy of Science* (Oxford: Oxford University Press, 1969), which is a monumental survey of the development of modern philosophy as a whole in the context of the science of the time, culminating in the work of Kant. There Buchdahl also presents his influential "looseness of fit" interpretation of the relationship between the *Metaphysical Foundations* and the *Critique of Pure Reason*, according to which the former work is constructed on analogy with the latter rather than being strictly and deductively derived from it. Gordon Brittan's *Kant's Theory of Science* (Princeton: Princeton University Press, 1978) is a detailed and important discussion of Kant's views from the perspective of twentieth-century Anglo-American philosophy of science. The anthology edited by Lewis White Beck, *Kant's Theory of Knowledge* (Dordrecht: Reidel, 1974), contains several important essays on Kant's philosophy of science, as does Allen Wood, ed., *Self and Nature in Kant's Philosophy* (Ithaca: Cornell University Press, 1984). The volume edited by Robert Butts, *Kant's Philosophy of Physical Science* (Dordrecht: Reidel, 1986), is wholly devoted to the philosophy of natural science and was assembled, in particular, to commemorate the 200th anniversary of the publication of the *Metaphysical Foundations*. My own *Kant and the Exact Sciences* (Cambridge, Mass.: Harvard University Press, 1992) builds on work originally appearing in the Butts volume. A more recent anthology, presenting a good cross-section, in particular, of the state of contemporary research in the field, is Eric Watkins, ed., *Kant and the Sciences* (Oxford: Oxford University Press, 2001), which is valuable not only for the essays

themselves but also for the extensive references to other contemporary literature contained therein.

There is a large and important literature devoted to the *Metaphysical Foundations* and related issues in other languages, principally German, extending back to the early nineteenth century. The most comprehensive of these works is Erich Adickes, *Kant als Naturforscher*, 2 vols. (Berlin: De Gruyter, 1924–25), which discusses in considerable detail not only the *Metaphysical Foundations* but also virtually all of Kant's work in natural science and natural philosophy. More recent works include G. Martin, *Immanuel Kant. Ontologie und Wissenschaftslehre* (Cologne: Kölner Universitätsverlag, 1951; translated as *Kant's Metaphysics and Theory of Science* [Manchester: Manchester University Press, 1955]); J. Vuillemin, *Physique et Métaphysique Kantienne* (Paris: Presses Universitaires de France, 1955); P. Plaass, *Kant's Theorie der Naturwissenschaft* (Göttingen: Vandenhoeck & Ruprecht, 1965; translated as *Kant's Theory of Natural Science* [Dordrecht: Kluwer, 1994]); L. Schäfer, *Kants Metaphysik der Natur* (Berlin: De Gruyter, 1966); H. Hoppe, *Kants Theorie der Physik* (Frankfurt: Klostermann, 1969); K. Gloy, *Die Kantische Theorie der Naturwissenschaft* (Berlin: De Gruyter, 1971); and K. Cramer, *Nicht-reine synthetische Urteile a priori* (Heidelberg: Carl Winter, 1985). Konstantin Pollok, *Kants "Metaphysische Anfangsgründe der Naturwissenschaft": ein kritischer Kommentar* (Hamburg: Felix Meiner, 2001), is an extremely detailed and up-to-date "critical commentary" to the *Metaphysical Foundations*, providing very useful information, in particular, about the contemporaneous context in which Kant was writing. Pollok's definitive modern edition, *Metaphysische Anfangsgründe der Naturwissenschaft* (Hamburg: Felix Meiner, 1997), also contains a valuable introduction by the editor, followed by extensive bibliographical information about both Kant's work itself (together with related materials concerning its contemporary reception) and the secondary literature up to 1997.

A further important topic, which has recently attracted significant interest, is the relationship between the *Metaphysical Foundations* and Kant's late unpublished reflections contained in the *Opus postumum*. This late work begins as an attempt to draft a *Transition from the Metaphysical Foundations of Natural Science to Physics*, where, in particular, some of the more empirical and physical topics considered in the General Remark to Dynamics are now taken up again – Kant now thinks, in fact, that precisely these more empirical aspects of natural science themselves

require an a priori foundation. So the relationship between the *Metaphysical Foundations* and the *Opus postumum* is of the greatest interest in connection with Kant's evolving thinking on the relationship, more generally, between the a priori and the empirical (a problem which first takes center stage in the *Critique of Judgement* in 1790). Recent discussions of this topic include Burkhardt Tuschling, *Metaphysische und transzendente Dynamik in Kants Opus postumum* (Berlin: De Gruyter, 1971); Vittorio Mathieu, *Kants Opus postumum* (Frankfurt: Klostermann, 1989); Part II of my *Kant and the Exact Sciences*; and Eckart Förster, *Kant's Final Synthesis* (Cambridge, Mass.: Harvard University Press, 2000). Förster has also translated and edited (substantial selections from) the *Opus postumum* (Cambridge: Cambridge University Press, 1993).

Note on texts and translation

The translation is based on the text in volume 4 of the Akademie Edition of *Kant's gesammelte Schriften* (Berlin: De Gruyter, 1902–), the page numbers of which appear in the margins. Where I have deviated from that text in favor of the original edition, I have appended an explanatory note. In general, references to Kant's works in the introduction and editor's notes are by volume and page number(s) of the Akademie Edition (as in Ak 4:476–77). The *Critique of Pure Reason*, as is usual, is cited by the pagination of the first (A) and/or second (B) editions. All translations from Kant's works used here are my own. The Cambridge Edition of the Works of Immanuel Kant, under the general editorship of Paul Guyer and Allen Wood, uses the Akademie Edition pagination in the margins, or, in the case of the *Critique of Pure Reason*, the pagination of the first and/or second editions.

In carrying out the translation I have consulted the two previous English renderings: those of E. Belfort Bax (London: George Bell & Sons, 1883) and James Ellington (Indianapolis: Bobbs-Merrill, 1970). Although he revises the earlier translation in several significant respects, Ellington still follows Bax rather closely. I have chosen, however, to deviate quite fundamentally from both Ellington and Bax, and to begin again from scratch. First, I have attempted to be as scrupulous as possible about respecting the nuances of Kant's technical terminology and phraseology – so that, in particular, Kant's conceptual distinctions are reflected as faithfully as possible. Second, I have also attempted, as far as possible, to render Kant's sentences into readable English – so as not to create additional problems for the reader in grappling with an already quite difficult text.

The present translation differs slightly from my contribution to the Cambridge Edition of the Works of Immanuel Kant, *Theoretical Philosophy after 1781*. In particular, I have made some small changes in the editor's notes (using the new translation of Newton's *Principia* by I. B. Cohen and Anne Whitman, for example), and, in the remark to the second proposition of the Mechanics (4:543), I have definitively settled on "continuity" for the translation of *Stetigkeit*.

Metaphysical Foundations of Natural Science

Preface

If the word nature is taken simply in its *formal* meaning, where it means the first inner principle of all that belongs to the existence of a thing,* then there can be as many different natural sciences as there are specifically different things, each of which must contain its own peculiar inner principle of the determinations belonging to its existence. But nature is also taken otherwise in its *material* meaning, not as a constitution,^a but as the sum total of all things, insofar as they can be *objects of our senses*, and thus also of experience. Nature, in this meaning, is therefore understood as the whole of all appearances, that is, the sensible world, excluding all nonsensible objects. Now nature, taken in this meaning of the word, has two principal parts, in accordance with the principal division of our senses, where the one contains the objects of the *outer* senses, the other the object of *inner* sense. In this meaning, therefore, a twofold doctrine of nature is possible, the *doctrine of body* and the *doctrine of the soul*, where the first considers *extended* nature, the second *thinking* nature.

Every doctrine that is supposed to be a system, that is, a whole of cognition ordered according to principles, is called a science. And, since such principles may be either principles of *empirical* or of *rational* connection of cognitions into a whole, then natural science, be it the doctrine of body or the doctrine of the soul, would have to be divided into *historical* or *rational* natural science, were it not that the word *nature* (since this

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* Essence is the first inner principle of all that belongs to the possibility of a thing. Therefore, one can attribute only an essence to geometrical figures, but not a nature (since in their concept nothing is thought that would express an existence).

^a *Beschaffenheit*.

signifies a derivation of the manifold belonging to the existence of things from their inner *principle*) makes necessary a cognition through reason of the interconnection of natural things, insofar as this cognition is to deserve the name of a science. Therefore, the doctrine of nature can be better divided into *historical doctrine of nature*, which contains nothing but systematically ordered facts about natural things (and would in turn consist of *natural description*, as a system of classification for natural things in accordance with their similarities, and *natural history*, as a systematic presentation of natural things at various times and places), and natural science. Natural science would now be either *properly* or *improperly* so-called natural science, where the first treats its object wholly according to a priori principles, the second according to laws of experience.

What can be called *proper* science is only that whose certainty is apodictic; cognition that can contain mere empirical certainty is only *knowledge*^b improperly so-called. Any whole of cognition that is systematic can, for this reason, already be called *science*, and, if the connection of cognition in this system is an interconnection of grounds and consequences, even *rational* science. If, however, the grounds or principles themselves are still in the end merely empirical, as in chemistry, for example, and the laws from which the given facts are explained through reason are mere laws of experience, then they carry with them no consciousness of their *necessity* (they are not apodictically certain), and thus the whole of cognition does not deserve the name of a science in the strict sense; chemistry should therefore be called a systematic art rather than a science.

A rational doctrine of nature thus deserves the name of a natural science, only in case the fundamental natural laws therein are cognized a priori, and are not mere laws of experience. One calls a cognition of nature of the first kind *pure*, but that of the second kind is called *applied* rational cognition. Since the word nature already carries with it the concept of laws, and the latter carries with it the concept of the *necessity* of all determinations of a thing belonging to its existence, one easily sees why natural science must derive the legitimacy of this title only from its pure part – namely, that [469] which contains the a priori principles of all other natural explanations – and why only in virtue of this pure part is natural science to be proper science. Likewise, [one sees] that, in accordance with demands of reason, every doctrine of nature must finally lead to natural science and conclude

^b *Wissen*. Cf. “science [*Wissenschaft*]” in the previous sentence.

there, because this necessity of laws is inseparably attached to the concept of nature, and therefore makes claim to be thoroughly comprehended. Hence, the most complete explanation of given appearances from chemical principles still always leaves behind a certain dissatisfaction, because one can adduce no a priori grounds for such principles, which, as contingent laws, have been learned merely from experience.

All *proper* natural science therefore requires a *pure* part, on which the apodictic certainty that reason seeks therein can be based. And because this pure part is wholly different, in regard to its principles, from those that are merely empirical, it is also of the greatest utility to expound this part as far as possible in its entirety, separated and wholly unmixed with the other part; indeed, in accordance with the nature of the case it is an unavoidable duty with respect to method. This is necessary in order that one may precisely determine what reason can accomplish for itself, and where its power begins to require the assistance of principles of experience. Pure rational cognition from mere *concepts* is called pure philosophy or metaphysics; by contrast, that which grounds its cognition only on the *construction* of concepts, by means of the presentation of the object in an a priori intuition, is called mathematics.

Properly so-called natural science presupposes, in the first place, metaphysics of nature. For laws, that is, principles of the necessity of that which belongs to the *existence* of a thing, are concerned with a concept that cannot be constructed, since existence cannot be presented a priori in any intuition. Thus proper natural science presupposes metaphysics of nature. Now this latter must always contain solely principles that are not empirical (for precisely this reason it bears the name of a metaphysics), but it can still either: *first*, treat the laws that make possible the concept of a nature in general, even without relation to any determinate object of experience, and thus undetermined with respect to the nature of this or that thing in the sensible world, in which case it is the *transcendental* part of the metaphysics of nature; or *second*, concern itself with a particular nature of this or that kind of thing, for which an empirical concept is given, but still in such a manner that, outside of what lies in this concept, no other empirical principle is used for its cognition (for example, it takes the empirical concept of matter or of a thinking being as its basis, and it seeks that sphere of cognition of which reason is capable a priori concerning these objects), and here such a science must still always be called a metaphysics of nature, namely, of corporeal or of thinking nature. However,

[in this second case] it is then not a general, but a *special* metaphysical natural science (physics or psychology), in which the above transcendental principles are applied to the two species of objects of our senses.¹

I assert, however, that in any special doctrine of nature there can be only as much *proper* science as there is *mathematics* therein. For, according to the preceding, proper science, and above all proper natural science, requires a pure part lying at the basis of the empirical part, and resting on a priori cognition of natural things. Now to cognize something a priori means to cognize it from its mere possibility. But the possibility of determinate natural things cannot be cognized from their mere concepts; for from these the possibility of the thought (that it does not contradict itself) can certainly be cognized, but not the possibility of the object, as a natural thing that can be given outside the thought (as existing). Hence, in order to cognize the possibility of determinate natural things, and thus to cognize them a priori, it is still required that the *intuition* corresponding to the concept be given a priori, that is, that the concept be constructed. Now rational cognition through construction of concepts is mathematical. Hence, although a pure philosophy of nature in general, that is, that which investigates only what constitutes the concept of a nature in general, may indeed be possible even without mathematics, a pure doctrine of nature concerning *determinate* natural things (doctrine of body or doctrine of soul) is only possible by means of mathematics. And, since in any doctrine of nature there is only as much proper science as there is a priori knowledge therein, a doctrine of nature will contain only as much proper science as there is mathematics capable of application there.

¹ See the discussion in the Architectonic of Pure Reason in the first *Critique*: “Metaphysics in the narrower sense consists of *transcendental philosophy* and the *physiology* of pure reason. The former considers only the *understanding* and reason itself in a system of concepts and principles that relate to objects in general, without assuming objects that may be *given* (*Ontologia*). The latter considers *nature* – i.e., the totality of *given* objects . . . and is therefore *physiology* (although only *rationalis*)” (A845/B873). After explaining that the latter doctrine (rational physiology) consists in turn of “metaphysics of corporeal nature” or “rational physics,” and “metaphysics of thinking nature” or “rational psychology” (A846/B874), Kant then continues as follows: “how can I expect an a priori cognition, and thus a metaphysics, of objects insofar as they are given to our senses, and therefore given a posteriori? . . . The answer is: we take no more from experience than what is necessary to *give* us an object – of either outer or inner sense. The former takes place through the mere concept of matter (impenetrable, lifeless extension), the latter through the concept of a thinking being (in the empirical inner representation: I think)” (A847–48/B875–76).

So long, therefore, as there is still for chemical actions of matters on one another no concept to be discovered that can be constructed, that is, no law of the approach or withdrawal of the parts of matter can be specified [471] according to which, perhaps in proportion to their density or the like, their motions and all the consequences thereof can be made intuitive and presented a priori in space (a demand that will only with great difficulty ever be fulfilled), then chemistry can be nothing more than a systematic art or experimental doctrine, but never a proper science, because its principles are merely empirical, and allow of no a priori presentation in intuition. Consequently, they do not in the least make the principles of chemical appearances conceivable with respect to their possibility, for they are not receptive to the application of mathematics.

Yet the empirical doctrine of the soul must remain even further from the rank of a properly so-called natural science than chemistry. In the first place, because mathematics is not applicable to the phenomena of inner sense and their laws, the only option one would have would be to take the *law of continuity* in the flux of inner changes into account – which, however, would be an extension of cognition standing to that which mathematics provides for the doctrine of body approximately as the doctrine of the properties of the straight line stands to the whole of geometry. For the pure inner intuition in which the appearances of the soul are supposed to be constructed is *time*, which has only one dimension. [In the second place,] however, the empirical doctrine of the soul can also never approach chemistry even as a systematic art of analysis or experimental doctrine, for in it the manifold of inner observation can be separated only by mere division in thought, and cannot then be held separate and recombined at will (but still less does another thinking subject suffer himself to be experimented upon to suit our purpose), and even observation by itself already changes and displaces the state of the observed object. Therefore, the empirical doctrine of the soul can never become anything more than an historical doctrine of nature, and, as such, a natural doctrine of inner sense which is as systematic as possible, that is, a natural description of the soul, but never a science of the soul, nor even, indeed, an experimental psychological doctrine. This is also the reason for our having used, in accordance with common custom, the general title of natural science for this work, which actually contains the principles of the doctrine of body, for only to it

does this title belong in the proper sense, and so no ambiguity is thereby produced.²

But in order to make possible the application of mathematics to the doctrine of body, which only through this can become natural science, principles for the *construction* of the concepts that belong to the possibility of matter in general must be introduced first. Therefore, a complete analysis of the concept of a matter in general will have to be taken as the basis, and this is a task for pure philosophy – which, for this purpose, makes use of no particular experiences, but only that which it finds in the isolated (although intrinsically empirical) concept itself, in relation to the pure intuitions in space and time, and in accordance with laws that already essentially attach to the concept of nature in general, and is therefore a genuine *metaphysics of corporeal nature*. [472]

Hence all natural philosophers who have wished to proceed mathematically in their occupation have always, and must have always, made use of metaphysical principles (albeit unconsciously), even if they themselves solemnly guarded against all claims of metaphysics upon their science. Undoubtedly they have understood by the latter the folly of contriving possibilities at will and playing with concepts, which can perhaps not be presented in intuition at all, and have no other certification of their objective reality than that they merely do not contradict themselves. All true metaphysics is drawn from the essence of the faculty of thinking itself, and is in no way fictitiously invented^c on account of not being borrowed from experience. Rather, it contains the pure actions of thought, and thus a priori concepts and principles, which first bring the manifold of *empirical representations* into the law-governed connection through which it can become *empirical cognition*, that is, experience. Thus these mathematical physicists could in no way avoid metaphysical principles, and, among them, also not those that make the concept of their proper object, namely, matter, a priori suitable for application to outer experience, such as the

^c *erdichtet*.

² See A381: “When we compare the *doctrine of the soul*, as the physiology of inner sense, with the *doctrine of body*, as a physiology of the objects of the outer senses, we find that, aside from the circumstance that much that is empirical can be cognized in both, there is still this remarkable difference: In the latter science much that is a priori can be synthetically cognized from the mere concept of an extended, impenetrable being, but in the former science nothing at all that is a priori can be synthetically cognized from the concept of a thinking being.” And compare the discussion of empirical psychology at A848–49/B876–77.

concept of motion, the filling of space, inertia, and so on.³ But they rightly held that to let merely empirical principles govern these concepts would in no way be appropriate to the apodictic certainty they wished their laws of nature to possess, so they preferred to postulate such [principles], without investigating them with regard to their a priori sources.

[473] Yet it is of the greatest importance to separate heterogeneous principles from one another, for the advantage of the sciences, and to place each in a special system so that it constitutes a science of its own kind, in order to guard against the uncertainty arising from mixing things together, where one finds it difficult to distinguish to which of the two the limitations, and even mistakes, that might occur in their use may be assigned. For this purpose I have considered it necessary [to isolate] the former from the pure part of natural science (*physica generalis*), where metaphysical and mathematical constructions customarily run together, and to present them, together with principles of the construction of these concepts (and thus principles of the possibility of a mathematical doctrine of nature itself), in a system.⁴ Aside from the already mentioned advantage that it provides, this isolation has also a special charm arising from the unity of cognition, when one takes care that the boundaries of the sciences do not run together, but rather each takes in its own separated field.

The following can serve as still another ground for commending this procedure. In everything that is called metaphysics one can hope for the *absolute completeness* of the sciences, of such a kind one may expect in no other type of cognition. Therefore, just as in the metaphysics of nature in general, here also the completeness of the metaphysics of corporeal nature can confidently be expected. The reason is that in metaphysics the object is

³ Compare the definition of matter cited in note 1 above (“impenetrable, lifeless, extension”) and the parallel discussion in §15 of the *Prolegomena* – which gives the relevant list of concepts as “the concept of *motion*, of *impenetrability* (on which the empirical concept of matter rests), of *inertia*, and others” (Ak 4:295). (Note that in the Remark to Proposition 3 of the *Mechanics*, “inertia” is equated with “lifelessness” [544].)

⁴ *Um deswillen habe ich für nöthig gehalten, von dem reinen Theile der Naturwissenschaft (physica generalis), wo metaphysische und mathematische Constructionen durch einander zu laufen pflegen, die erstere und mit ihnen zugleich die Prinzipien der Construction dieser Begriffe, also der Möglichkeit einer mathematischen Naturlehre selbst, in einem System darzustellen.* This difficult sentence has led to considerable controversy. Plaass (1965) and Schäfer (1966) have made the notion of “metaphysical construction” central to their interpretations, whereas Hoppe (1969) and Gloy (1976) have suggested that “concepts” or “principles” should follow “metaphysical” in the sentence. Here, in any case, one should compare the section on the Discipline of Pure Reason in its Dogmatic Employment from the Doctrine of Method of the first *Critique* (A712–38/B740–66) – which certainly suggests that the construction of concepts is precisely what *distinguishes* mathematics from philosophy.

only considered in accordance with the general laws of thought, whereas in other sciences it must be represented in accordance with data of intuition (pure as well as empirical), where the former, because here the object has to be compared always with *all* the necessary laws of thought, must yield a determinate number of cognitions that may be completely exhausted, but the latter, because they offer an infinite manifold of intuitions (pure or empirical), and thus an infinite manifold of objects of thought, never attain absolute completeness, but can always be extended to infinity, as in pure mathematics and empirical doctrine of nature. I also take myself to have completely exhausted this metaphysical doctrine of body, so far as it may extend, but not to have thereby accomplished any great [piece of] work.

But the schema for completeness of a metaphysical system, whether it be of nature in general, or of corporeal nature in particular, is the table of categories.[†] For there are no more pure concepts of the understanding [474]

[†] In the *Allgemeine Literatur Zeitung*, No. 295, in the review of *Institutiones Logicae et Metaphysicae* by Prof. Ulrich, I find doubts, which are not directed against this table of pure concepts of the understanding, but rather against the inferences drawn therefrom to the determination of the limits of the entire faculty of pure reason, and thus all metaphysics, [doubts] with respect to which the deeply delving reviewer declares himself to be in agreement with the no less penetrating author. And, in fact, since these doubts are supposed to concern precisely the principal basis of my system articulated in the *Critique*, they would be grounds for thinking that this system, with respect to its principal aim, does not come close to carrying that apodictic conviction that is required for eliciting an unqualified acceptance. This principal basis is said to be the *deduction* of the pure concepts of the understanding, which is expounded partly in the *Critique* and partly in the *Prolegomena*, and which, however, in the part of the *Critique* that ought to be precisely the most clear, is rather the most obscure, or even revolves in a circle, etc. I direct my reply to these objections only to their principal point, namely, the claim that *without an entirely clear and sufficient deduction of the categories* the system of the *Critique of Pure Reason* totters on its foundation. I assert, on the contrary, that the system of the *Critique* must carry apodictic certainty for whoever subscribes (as the reviewer does) to my propositions concerning the sensible character of all our intuition, and the adequacy of the table of categories, as determinations of our consciousness derived from the logical functions in judgments in general, because it is erected upon the proposition *that the entire speculative use of our reason never reaches further than to objects of possible experience*. For if we can prove that the categories which reason must use in all its cognition can have no other use at all, except solely in relation to objects of possible experience (insofar as they simply make possible the form of thought in such experience), then, although the answer to the question *how* the categories make such experience possible is important enough for *completing* the deduction where possible, with respect to the principal end of the system, namely, the determination of the limits of pure reason, it is in no way *compulsory*, but merely *meritorious*. For the deduction is already carried *far enough* for this purpose if it shows that categories of thought are nothing but mere forms of judgments insofar as they are applied to intuitions (which for us are always sensible), and that they thereby first of all obtain objects and become cognitions; because this already suffices to ground with complete certainty the entire system of the *Critique* properly speaking. Thus Newton's system of universal gravitation stands firm, even though it involves the difficulty that one cannot explain how attraction at a distance is possible; but *difficulties are not doubts*. That the above fundamental basis

[475] which can be concerned with the nature of things. All determinations of the general concept of a matter in general must be able to be brought under the four classes of [pure concepts of the understanding], those of *quantity*, of *quality*, of *relation*, and finally of *modality* – and so, too, [must]
 [476] all that may be either thought a priori in this concept, or presented in

stands firm, even without a complete deduction of the categories, I now prove from the following granted propositions:

[475] 1. *Granted*: that the table of categories contains all pure concepts of the understanding, just as it contains all formal actions of the understanding in judging, from which the concepts of the understanding are derived, and from which they differ only in that, through the concepts of the understanding, an object is thought as *determined* with respect to one or another function of judgment. (Thus, for example, in the categorical judgment *the stone is hard*, the *stone* is used as subject, and *hard* as predicate, in such a way that the understanding is still free to exchange the logical function of these concepts, and to say that something hard is a stone. By contrast, if I represent it to myself as *determined in the object* that the stone must be thought only as subject, but hardness only as predicate, in any possible determination of an object (not of the mere concept), then the very same logical functions now become *pure concepts of the understanding* of objects, namely, as *substance* and *accident*.)

2. *Granted*: that the understanding by its nature contains synthetic a priori principles, through which it subjects all objects that may be given to it to these categories, and, therefore, there must also be intuitions given a priori that contain the conditions required for the application of these pure concepts of the understanding, because *without intuition there can be no object*, with respect to which the logical function could be determined as category, and thus no cognition of any object whatsoever, and hence without pure intuition no principle that determines it a priori for this purpose.

3. *Granted*: that these pure intuitions can never be anything other than mere forms of the *appearances* of outer or of inner sense (space and time), and therefore of the *objects of possible experience* alone.

It then follows: that all use of pure reason can never extend to anything other than objects of experience, and, since nothing empirical can be the condition of a priori principles, the latter can be nothing more than principles of the *possibility of experience* in general. This alone is the true and sufficient basis for the determination of the limits of pure reason, but not the solution to the problem *how* experience is now possible by means of these categories, and only through these categories alone. The latter problem, although without it the structure still stands firm, has great importance nonetheless, and, as I now understand it, [it can be solved with] just as much ease, since it can almost be accomplished through a single inference from the precisely determined definition of a *judgment* in general (an action through which given representations first become cognitions of an object). The obscurity that attaches to my earlier discussions in this part of the deduction (and which I do not deny), is to be attributed to the common fortunes of the understanding in its investigations, in which the shortest way is commonly not the first way that it becomes aware of. Therefore, I shall take up the next opportunity to make up for this deficiency (which concerns only the manner of presentation, and not the ground of explanation, which is already stated correctly there), so that the perceptive reviewer may not be left with the necessity, certainly unwelcome even to himself, of taking refuge in a preestablished harmony to explain the surprising agreement of appearances with the laws of the understanding, despite their having entirely different sources from the former. This remedy would be much worse than the evil it is supposed to cure, and, on the contrary, actually cannot help at all. For the *objective necessity* that characterizes the pure concepts of the understanding (and the principles of their application to appearances), in the concept of cause in connection with the effect, for example, is still not forthcoming. Rather, it all remains only *subjectively necessary*, but objectively merely contingent, placing together, precisely as Hume has it

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mathematical construction, or given as a determinate object of experience. There is no more to be done, or to be discovered, or to be added here, except, if need be, to improve it where it may lack in clarity or exactitude.^d

The concept of matter had therefore to be carried through all four of the indicated functions of the concepts of the understanding (in four chapters), where in each a new determination of this concept was added. The basic determination of something that is to be an object of the outer senses had to be motion, because only thereby can these senses be affected. The understanding traces back all other predicates of matter belonging to its nature to this, and so natural science, therefore, is either a pure or applied *doctrine of motion*. The *metaphysical* foundations of natural science are therefore to be brought under *four* chapters. The *first* considers *motion* as a pure *quantum* in accordance with its composition, without any quality of the movable, and may be called **phoronomy**. The *second* takes into consideration motion as belonging to the *quality* of matter, under the name of an original moving force, and is therefore called **dynamics**. The *third* considers matter with this quality as in *relation* to another through its own inherent motion, and therefore appears under the name of **mechanics**. The *fourth* chapter, however, determines matter's motion or rest merely in relation to the mode of representation or *modality*, and thus as appearance of the outer senses, and is called **phenomenology**. [477]

Yet aside from the inner necessity to isolate the metaphysical foundations of the doctrine of body, not only from physics, which needs empirical principles, but even from the rational premises of physics that concern the use of mathematics therein, there is still an external, certainly only accidental, but nonetheless important reason for detaching its detailed treatment from the general system of metaphysics, and presenting it systematically as a special whole. For if it is permissible to draw the boundaries of a science, not simply according to the constitution of the object and its specific mode of cognition, but also according to the end that one

when he calls this mere illusion from custom. No system in the world can derive this necessity from anywhere else than the principles lying a priori at the basis of the possibility of *thinking itself*, through which alone the cognition of objects whose appearance is given to us, that is, experience, becomes possible. Even if we suppose, therefore, that the explanation of *how* experience thereby becomes possible in the first place could never be sufficiently carried out, it still remains incontrovertibly certain *that* it is possible solely through these concepts, and, conversely, that these concepts are capable of meaning and use in no other relation than to objects of experience.

^d *Gründlichkeit*.

has in mind for this science itself in uses elsewhere; and if one finds that metaphysics has busied so many heads until now, and will continue to do so, not in order thereby to extend natural knowledge (which takes place much more easily and surely through observation, experiment, and the application of mathematics to outer appearances), but rather so as to attain cognition of that which lies wholly beyond all boundaries of experience, of God, Freedom, and Immortality; then one gains in the advancement of this goal if one frees it^e from an offshoot that certainly springs from its root, but nonetheless only hinders its regular growth, and one plants this offshoot specially, yet without failing to appreciate the origin of [this offshoot] from it,^f and without omitting the mature plant from the system of general metaphysics. This does not impair the completeness of general metaphysics, and in fact facilitates the uniform progress of this science [478] towards its end, if, in all instances where one requires the general doctrine of body, one may call only upon the isolated system, without swelling this greater system with the latter. It is also indeed very remarkable (but cannot be expounded in detail here)⁵ that general metaphysics, in all instances where it requires examples (intuitions) in order to provide meaning for its pure concepts of the understanding, must always take them from the general doctrine of body, and thus from the form and principles of outer intuition; and, if these are not exhibited completely, it gropes uncertainly and unsteadily among mere meaningless concepts. This is the source of the well-known disputes, or at least obscurity, in the questions concerning the possibility of a conflict of realities, of intensive magnitude, and so on, in which the understanding is taught only by examples from corporeal nature what the conditions are under which such concepts can alone have objective reality, that is, meaning and truth. And so a separated metaphysics of corporeal nature does excellent and indispensable service for *general* metaphysics, in that the former furnishes examples (instances *in concreto*) in which to realize the concepts and propositions of the latter (properly speaking, transcendental philosophy), that is, to give a mere form of thought sense and meaning.

^e *sic*. The reference is most likely to “metaphysics” – or possibly to “this science.”

^f *jener*. See note e above.

⁵ See the General Remark to the System of Principles in the second edition of the *Critique*: “But it is even more remarkable that, in order to understand the possibility of things in conformity with the categories, and thus to verify the *objective reality* of the latter, we require not merely intuitions, but always even *outer intuitions*” (B291).

In this treatise, although I have not followed the mathematical method with thoroughgoing rigor (which would have required more time than I had to spend thereon), I have nonetheless imitated that method⁶ – not in order to obtain a better reception for the treatise, through an ostentatious display of exactitude,^g but rather because I believe that such a system would certainly be capable of this rigor, and also that such perfection could certainly be reached in time by a more adept hand, if, stimulated by this sketch, mathematical natural scientists should find it not unimportant to treat the metaphysical part, which they cannot leave out in any case, as a special fundamental part in their general physics, and to bring it into union with the mathematical doctrine of motion.

Newton, in the preface to his *Mathematical First Principles of Natural Science*, says (after he had remarked that geometry requires only two of the mechanical operations that it postulates, namely, to describe a straight line and a circle): *Geometry is proud of the fact that with so little derived from without it is able to produce so much.*^{‡7} By contrast, one can say of metaphysics: *it is dismayed that with so much offered to it by pure mathematics it can still accomplish so little.* Nevertheless, this small amount is still something that even mathematics unavoidably requires in its application to natural science; and thus, since it must here necessarily borrow from metaphysics, need also not be ashamed to let itself be seen in community with the latter. [479]

[‡] “Geometry can boast that with so few principles obtained from other fields, it can do so much.”
Newton Princ. Phil. Nat. Math. praefat.

^g *Gründlichkeit.*

⁶ Again, one should compare A726/B754 from the section of the first *Critique* cited in note 4 above, where Kant explicitly says that philosophy *cannot* imitate mathematics (*Meßkunst*). See also A735/B763.

⁷ Kant quotes the original Latin in the note: Gloriatur Geometria, quod tam paucis principiis aliunde petitis tam multa praestet. The quotation given is from I. Newton, *The Principia: Mathematical Principles of Natural Philosophy*, trans. I. B. Cohen and A. Whitman (Berkeley: University of California Press, 1999), p. 382.

First Chapter

Metaphysical Foundations of Phoronomy

Explication⁸ 1

Matter is the *movable* in space. That space which is itself movable is called material, or also *relative space*; that in which all *motion* must finally be thought (and which is therefore itself absolutely^a immovable) is called pure, or also *absolute space*.

Remark 1

Since in phoronomy nothing is to be at issue except motion, no other property is here ascribed to the *subject* of motion, namely, matter, aside from *movability*. It can itself so far, therefore, also be considered as a point, and one abstracts in phoronomy from all inner constitution, and therefore also from the quantity of the movable, and concerns oneself only with motion and what can be considered as quantity in motion (speed and direction).⁹ – If the expression “body” should nevertheless sometimes be used here, this is only to anticipate to some extent the application of the principles of phoronomy to the more determinate concepts of matter that are still to follow, so that the exposition may be less abstract and more comprehensible.

^a *schlechterdings*.

⁸ *Erklärung*. For the distinction between mathematical *definitions* (**Definitionen**) and philosophical *explications* (**Erklärungen**) see A727–32/B755–60.

⁹ *Geschwindigkeit und Richtung*. Since Kant thus explicitly distinguishes these two elements, it is clear that by *Geschwindigkeit* he means what we now refer to as the scalar quantity speed as opposed to the vector quantity velocity.

Remark 2

[481]

If I am to explicate the concept of matter, not through a predicate that belongs to it itself as object, but only by relation to that cognitive faculty in which the representation can first of all be given to me, then every *object of the outer senses* is matter, and this would be the merely metaphysical explication thereof. Space, however, would be merely the form of all outer sensible intuition (we here leave completely aside the question whether just this form also belongs *in itself* to the outer object we call matter, or remains only in the constitution of our sense). *Matter*, as opposed to *form*, would be that in the outer intuition which is an object of sensation, and thus the properly empirical element of sensible and outer intuition, because it can in no way be given a priori. In all experience something must be sensed, and that is the real of sensible intuition, and therefore the space, in which we are to arrange our experience of motion, must also be sensible – that is, it must be designated^b through what can be sensed – and this, as the totality of all objects of experience, and itself an object of experience, is called *empirical space*. But this, as material, is itself movable. But a movable space, if its motion is to be capable of being perceived, presupposes in turn an enlarged material space, in which it is movable; this latter presupposes in precisely the same way yet another; and so on to infinity.

Thus all motion that is an object of experience is merely relative; and the space in which it is perceived is a relative space, which itself moves in turn in an enlarged space, perhaps in the opposite direction, so that matter moved with respect to the first can be called at rest in relation to the second space, and these variations in the concept of motions progress to infinity along with the change of relative space. To assume an absolute space, that is, one such that, because it is not material, it can also not be an object of experience, as *given in itself*, is to assume something, which can be perceived neither in itself nor in its consequences (motion in absolute space), for the sake of the possibility of experience – which, however, must always be arranged without it. Absolute space is thus *in itself* nothing, and no object at all, but rather signifies^c only any other relative space, which I can always think beyond the given space, and which I can only defer to infinity beyond any given space, so as to include it and suppose it to be

^b bezeichnet. ^c bedeutet.

moved. Since I have the enlarged, although still always material, space only in thought, and since nothing is known to me of the matter that designates it, I abstract from the latter, and it is therefore represented [482] as a pure, nonempirical, and absolute space, with which I compare any empirical space, and in which I can represent the latter as movable (so that the enlarged space always counts as immovable). To make this into an actual thing is to transform the *logical universality* of any space with which I can compare any empirical space, as included therein, into a *physical universality* of actual extent, and to misunderstand reason in its idea.¹⁰

Finally, I further remark that, since the *movability* of an object in space cannot be cognized a priori, and without instruction through experience, I could not, for precisely this reason, enumerate it under the pure concepts of the understanding in the *Critique of Pure Reason*;¹¹ and that this concept, as empirical, could only find a place in a natural science, as applied metaphysics, which concerns itself with a concept given through experience, although in accordance with a priori principles.

Explication 2

Motion of a thing is the *change of its outer relations* to a given space.

Remark 1

I have so far placed the concept of motion at the basis of the concept of matter. For, since I wanted to determine this concept independently of the concept of extension, and could therefore consider matter also in a point, I could allow the common explication of *motion as change of place* to be used. Now, since the concept of a matter is to be explicated generally, and therefore as befitting also moving bodies, this definition is no longer sufficient. For the place of any body is a point. If one wants to determine the distance of the moon from the earth, one wants to know the distance between their places, and for this purpose one does not measure from an arbitrary point of the surface or interior of the earth to any arbitrary point of the moon, but chooses the shortest line from the central point of the

¹⁰ The notion of absolute space is further discussed in the General Remark to Phenomenology, where, in particular, it is characterized as a “necessary concept of reason” or “mere idea” [559].

¹¹ Compare A41/B58; see also, from the second edition, B155n.

one to the central point of the other, so that for each of these bodies there is only one point constituting its place. Now a body can move without changing its place, as in the case of the earth rotating around its axis. But its relation to external space still changes thereby; since it turns, for example, its different sides towards the moon in 24 hours, from which all kinds of varying effects then follow on the earth. Only of a movable, that is, physical, *point* can one say that motion is always change of place. One could object to this explication by pointing out that inner motion, fermentation, for example, is not included; but the thing one calls moving [483] must to that extent be considered as a unity. For example, that matter, as a *cask of beer*, is moved, means something different from the *beer in the cask* being in motion. The motion of a thing is not the same as motion in this thing, but here we are concerned only with the former case. But the application of this concept to the second case is then easy.

Remark 2

Motions can be either *rotating* (without change of place) or progressive, and the latter can be either motions that enlarge the space, or are limited to a given space. Of the *first* kind are the rectilinear motions, and also the curvilinear motions that do *not return* on themselves. Of the *second* [kind] are the motions that *return* on themselves. The latter in turn are either *circulating* or *oscillating*, that is, either circular or oscillatory motions. The former always traverse precisely the same space in the same direction, the latter always alternately back in the opposite direction, as in the case of oscillating pendulums. Belonging to both is still *tremor* (motus tremulus), which is not the progressive motion of a body, but nonetheless a reciprocating motion of a matter, which does not thereby change its place as a whole, as in the vibrations of a struck bell, or the tremors of the air set in motion by the sound. I only make mention of these various kinds of motion in phronomy, because one commonly uses the word *speed*, in the case of all nonprogressive [motions], in another meaning than in the case of the progressive ones, as the following remark shows.

Remark 3

In every motion direction and speed are the two moments for considering motion, if one abstracts from all other properties of the movable. I here

presuppose the usual definitions of both, but that of direction still requires various qualifications. A body moving in a circle changes its direction continuously, in such a way that it follows all possible directions in a surface along its way back to the point from which it set off, and yet one says that it moves always in the same direction, for example, a planet from west to east.^d

[484] But what is here the *side* towards which the motion is directed? This has a kinship with the following question: On what rests the inner difference of snails, which are otherwise similar and even equal, but among which one species is wound rightward, the other leftward; or the winding of the kidney bean and the hop, where the first runs around its pole like a corkscrew, or, as sailors would express it, *against the sun*, whereas the second runs *with the sun*? This is a concept which can certainly be constructed, but, as a concept, can in no way be made clear in itself by means of universal characteristics and in the discursive mode of cognition, and can yield no thinkable difference in the inner consequences in the things themselves (for example, in the case of those unusual people where all parts on dissection were found in agreement physiologically with other humans, except that all organs were transposed leftward or rightward contrary to the usual order), but is nevertheless a genuine mathematical, and indeed inner difference, which is connected with, although not identical to, the difference between two circular motions that are otherwise equal in all parts, but differ in direction. I have shown elsewhere¹² that, since this difference can certainly be given in intuition, but can in no way be captured in clear concepts, and thus cannot be rationally explicated (dari, non intelligi), it supplies a good confirming ground of proof for the proposition that space in general does not belong to the properties or relations of *things in themselves*, which would necessarily have to be reducible to objective concepts,^e but rather belongs merely to the subjective form of our sensible intuition of things or relations, which must remain completely unknown to us as to what they may be in themselves. Yet this is a digression from our present business, in which we must necessarily treat space as a *property* of the things under consideration, namely, corporeal beings, because these things are themselves only appearances of the outer senses, and only require to be explicated as such here. As far as

^d von Abend gegen Morgan. ^e die sich notwendig auf objective Begriffe müßten bringen lassen.

¹² See *Prolegomena to Any Future Metaphysics*, §13 (Ak 4:285–86).

the concept of speed is concerned, this expression sometimes acquires in use a deviant meaning. We say that the earth rotates faster around its axis than the sun, because it does this in a shorter time, although the motion of the latter is much faster. The circulation of the blood in a small bird is much faster than that in a human being, although its flowing motion in the first case has without any doubt less speed, and so also in the case of vibrations in elastic materials. The brevity of the time of return, whether it be circulating or oscillating motion, constitutes the basis for this usage, which, so long as one avoids misunderstanding, is also not incorrect. For this mere increase in the rapidity of the return, without increase in spatial speed, has its own very important effects in nature, concerning which, in the circulation of fluids in animals, perhaps not enough notice has yet been taken. In phoronomy we use the word “speed” purely in a spatial meaning $C = S/T$.¹³

Explication 3

[485]

Rest is perduring presence^f (*praesentia perdurabilis*) at the same place; what is *perduring* is that which exists throughout a time, that is, endures.

Remark

A body in motion is at every point of the line that it traverses for a moment. The question is now whether it rests there or moves. Without a doubt one will say the latter; for it is present at this point only insofar as it moves. Assume, however, that the motion of the body is such: $\begin{matrix} A & & B \\ 0 & \text{---} & 0 \end{matrix} \cdot \cdot \begin{matrix} a \\ 0 \end{matrix}$, that the body travels along the line AB with uniform speed forwards and backwards from B to A, and that, since the moment when it is at B is common to both motions, the motion from A to B is traversed in $\frac{1}{2}$ sec., that from B to A also in $\frac{1}{2}$ sec., and both together in one whole second, so that not even the smallest part of the time pertains to the presence of the body at B; then, without the least increase of these motions, the latter, having taken place in the direction BA, can be transformed into that in the direction Ba, lying in a straight line with AB, in which case the body, when it is at B, must be viewed as not at rest there, but as

^f beharrliche Gegenwart.

¹³ That is, *Celeritas est Spatium per Temporum* (speed equals distance over time).

moved. It would therefore have to be also viewed as moved at the point B in the first case of motion returning back on itself – which, however, is impossible; since according to what was assumed, this point comprises only a moment – belonging to the motion AB and simultaneously to the motion BA, which is opposite to AB and joined to AB in one and the same moment – of complete lack of motion. Therefore, if this constituted the concept of rest, then rest of the body would also have to be manifested in the uniform motion Aa at every point, for example, at B, which contradicts the above assertion. Suppose, however, that one imagines the line AB as erected above the point A, so that a body rising from A to B falls back again from B to A after it has lost its motion at B through gravity; I then ask whether the body at B can be viewed as moved or as at rest. Without a doubt one will say at rest; for all previous motion has been taken from it once it has reached this point, and after this an equivalent motion back is about to follow, and thus is not yet there; but the lack of motion, one will add, is rest. Yet in the first case of an assumed uniform motion, the motion BA could also not take place except through the fact that the motion AB [486] had previously ceased, and that from B to A was not yet there, so that a lack of all motion at B had to be assumed, and, according to the usual explication, also rest – but one may not assume it, because no body at a point of its uniform motion at a given speed can be thought of as at rest. So on what is based the appropriateness of the concept of rest in the second case, where the rising and falling are likewise separated from one another only by a moment? The reason for this lies in the fact that the latter motion is not thought of as uniform at a given speed, but rather first as uniformly decelerated and thereafter as uniformly accelerated,¹⁴ so that the speed at point B is not completely diminished, but only to a degree that is smaller than any given speed. With this [speed], therefore, the body, if it were to be viewed always as still rising, so that instead of falling back the line of its fall BA were to be erected in the direction Ba, would uniformly traverse, with a mere moment of speed (the resistance of gravity here being set aside), a space smaller than any given space in any given time, however large, and thus would in no way change its place (for

¹⁴ According to Galileo's law of free fall, an object thrown upward is uniformly decelerated on its upward trajectory and uniformly accelerated on its downward trajectory. In both cases, then, we have $v = gt$, where g is the constant of gravity. Therefore, v continuously decreases to zero and then continuously increases from zero.

any possible experience) in all eternity. It is therefore put into the state of an *enduring* presence at the same place, that is, a state of rest, even though, because of the continual influence of gravity, that is, the change of this state, it is immediately destroyed. To be in a *perduring state* and to *perdure in this state* (if nothing else displaces it) are two different, although not incompatible, concepts. Thus rest cannot be explicated as lack of motion, which, as $= 0$, can in no way be constructed, but must rather be explicated as *perduring* presence at the same place, since this concept can also be constructed, through the representation of a motion with infinitely small speed throughout a finite time, and can therefore be used for the ensuing application of mathematics to natural science.

Explication 4

To **construct** the concept of a *composite motion* means to present a motion a priori in intuition, insofar as it arises from two or more given motions united in one movable.

Remark

It is required for the construction of concepts that the conditions of their presentation not be borrowed from experience, and thus not presuppose certain forces whose existence can only be derived from experience; or, [487] in general, that the condition of the construction must not itself be a concept that can by no means be given a priori in intuition, such as, for example, the concept of cause and effect, action and resistance, etc. Now here it is above all to be noted that phoronomy has first to determine the construction of motions in general as *quantities*, and, since it has matter merely as *something movable* as its object, in which no attention at all is therefore paid to its quantity, [it has to determine] these motions a priori solely as quantities, with respect to both their speed and direction, and, indeed, with respect to their composition. For so much must be constituted wholly a priori, and indeed intuitively, on behalf of applied mathematics. For the rules for the connection of motions by means of physical causes, that is, forces, can never be rigorously expounded, until the principles of their composition in general have been previously laid down, purely mathematically, as basis.

Principle

Every motion, as object of a possible experience, can be viewed arbitrarily as motion of the body in a space at rest, or else as rest of the body, and, instead, as motion of the space in the opposite direction with the same speed.

Remark

To make the motion of a body into an experience, it is required that not only the body, but also the space in which it moves, be objects of outer experience, and thus material. Hence an absolute motion, that is, [one] in relation to a nonmaterial space, cannot be experienced at all, and thus is nothing for us (even if one wanted to grant that absolute space were something in itself). But in all relative motion the space itself, since it is assumed to be material, can in turn be represented as either at rest or as moved. The first occurs when, beyond the space in which I view the body as moved, no further enlarged space is given to me that includes it (as when I see a ball moving on the table in the cabin of a ship); the second occurs when, beyond the given space, another space that includes it is given (in the example mentioned, the bank of the river), since I can then, in relation to the latter, view the nearest space (the cabin) as moved, and [488] the body itself as possibly at rest. Now because it is completely impossible to determine for an empirically given space, no matter how enlarged it may be, whether it may or may not be moved in turn, in relation to an inclusive space of still greater extent, it must then be completely the same for all experience, and every consequence of experience, whether I wish to view a body as moved, or as at rest, but the space as moved in the opposite direction with the same speed. Further, since absolute space is nothing for all possible experience, the concepts are also the same whether I say that a body moves in relation to this given space, in such and such direction with such and such speed, or I wish to think the body as at rest, and to ascribe all this, but in the opposite direction, to the space. For any concept is entirely the same as a concept whose differences from it have no possible example at all, being only different with respect to the connection we wish to give it in the understanding.

We are also incapable, in any experience at all, of assigning a fixed point in relation to which it would be determined what motion and rest are to

be absolutely; for everything given to us in this way is material, and thus movable, and (since we are acquainted with no outermost limit of possible experience in space) is perhaps also actually moved, without our being able to perceive this motion. – Of this motion of a body in empirical space, I can give a part of the given speed to the body, and the other to the space, but in the opposite direction, and the whole possible experience, with respect to the consequences of these two combined motions, is entirely the same as that experience in which I think the body as alone moved with the whole speed, or the body as at rest and the space as moved with the same speed in the opposite direction. *But here I assume all motions to be rectilinear.* For in regard to curvilinear motions, it is not in all respects the same whether I am authorized to view the body (the earth in its daily rotation, for example) as moved and the surrounding space (the starry heavens) as at rest, or the latter as moved and the former as at rest, which will be specially treated in what follows. Thus in phoronomy, where I consider the motion of a body only in relation to the space (which has no influence at all on the rest or motion of the body), it is completely undetermined and arbitrary how much speed, if any, I wish to ascribe to the one or to the other. Later, in mechanics, where a moving body is to be considered in active relation to other bodies in the space of its motion, this will no longer be entirely the same, as will be shown in the proper place.¹⁵

Explication 5

[489]

The *composition of motion* is the representation of the motion of a point as the same as two or more motions of [this point] combined together.

Remark

In phoronomy, since I am acquainted with matter through no other property but its movability, and may thus consider it only as a point, motion can only be considered as the *describing of a space* – in such a way, however, that I attend not solely, as in geometry, to the space described, but also

¹⁵ See, in particular, the footnote to the Proof of Proposition 4 of the Mechanics chapter [547].

to the time in which, and thus to the speed with which, a point describes the space. Phoronomy is thus the pure theory of quantity (mathesis) of motions. The determinate concept of a quantity is the concept of the generation of the representation of an object through the composition of the homogeneous.¹⁶ Now since nothing is homogeneous with motion except motion in turn, phoronomy is a doctrine of the composition of the motions of one and the same point in accordance with its speed and direction, that is, the representation of a single motion as one that contains two or more motions at the same time, or two motions of precisely the same point at the same time, insofar as they constitute one motion *combined* – that is, [they] are the same as the latter, and do not, for example, produce it, as causes produce their effect. In order to find the motion arising from the composition of several motions, as many as one wishes, one need only, as in all generation of quantity, first seek for that motion which, under the given conditions, is composed from *two* motions; this is then combined with a third; and so on. Therefore, the doctrine of the composition of all motions can be reduced to that of two. But two motions of one and the same point, which are found there at the same time, can be distinguished in two ways, and, as such, can be combined there in three ways. First, they occur either in one and the *same line*, or in *different lines* at the same time; the latter are motions comprising an angle. Those occurring in one and the *same line* are now, with respect to direction, either *opposite* to one another or have the *same direction*. Because all these motions are considered as occurring at the same time, the ratio of speed results immediately from the ratio of the lines, that is, from the described spaces of the motion in the same time. There are therefore three cases: (1) The *two motions* (they may have equal or unequal speeds) are to constitute a composite motion combined in one body in the same direction. (2) The *two motions* of the same point (of equal or unequal speed), combined in opposite directions, are to constitute, through their composition, a third motion in the same line. (3) The two motions of a point are considered as

[490]

¹⁶ Compare the first paragraph of the Axioms of Intuition in the second edition: “[Appearances] can be apprehended in no other way . . . except through the synthesis of the manifold whereby the representation of a determinate space or time is generated, i.e., through the composition of the homogeneous and the consciousness of the synthetic unity of this (homogeneous) manifold. Now the consciousness of the manifold [of] homogeneous [elements] in intuition in general, insofar as the representation of an object first becomes possible, is the concept of quantity (*quanti*)” (B202–03).

composed with equal or unequal speeds, but in different lines comprising an angle.

Proposition

The composition of two motions of one and the same point can only be thought in such a way that one of them is represented in absolute space, and, instead of the other, a motion of the relative space with the same speed occurring in the opposite direction is represented as the same as the latter.

Proof

First case. Two motions in *one and the same line and direction* belong to one and the same point.

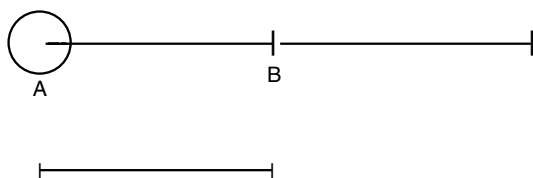


Figure 1

Two speeds AB and ab are to be represented as contained in one speed of motion. If one assumes these speeds to be equal for the moment, so that $AB = ab$, then I say that they cannot be represented at the same time in one and the same space (whether absolute or relative) in one and the same point. For, since the lines AB and ab designating the speeds are, properly speaking, the spaces they traverse in equal times, then the composition of these spaces AB and $ab = BC$, and hence the line AC as the sum of the spaces, would have to express the sum of the two speeds. But neither the part AB nor the part BC represents the speed $= ab$, for they are not traversed in the same time as ab . Therefore, the doubled line AC , traversed in the same time as the line ab , does not represent the twofold speed of the latter, which, however, was required. Therefore, the composition of two speeds in one direction cannot be represented intuitively *in the same space*.

[491] By contrast, if the body A with speed AB is represented as moved in absolute space, and, moreover, I give to the relative space a speed $ab = AB$ in the opposite direction $ba = CB$, then this is precisely the same as if I had imparted the latter speed to the body in the direction AB (Principle). The body then moves through the sum of the lines AB and $BC = 2ab$ in the same time in which it would have traversed the line $ab = AB$ alone, and its speed is thus represented as the sum of the two equal speeds AB and ab, which is what was required.

Second case. Two motions are to be combined in *precisely opposite directions* in one and the same point.

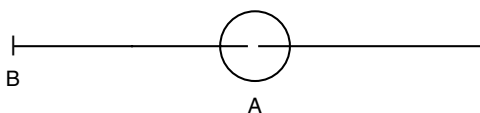


Figure 2

Let AB be one of these motions and AC the other in the opposite direction, whose speed we will assume here as equal to the first. Then even the thought of representing two such motions at the same time in exactly the same point within one and the same space would be impossible, and thus so would the case of such a composition of motions itself, which is contrary to the presupposition.

By contrast, think instead the motion AB in absolute space, but, instead of the motion AC in the same absolute space, the opposite motion CA of the relative space with exactly the same speed, which (according to the Principle) counts as entirely the same as the motion AC, and can therefore be posited wholly in place of the latter. Then two precisely opposite and equal motions of the same point at the same time can perfectly well be represented. Because the relative space now moves with the same speed $CA = AB$ in the same direction as the point A, this point, or the body found there, does not change its place in relation to the relative space. That is, a body moved in two exactly opposite directions with the same speed is at rest, or, expressed in general: its motion is equal to the difference of the speeds in the direction of the greater (which can easily be concluded from what has been proved).

[492] *Third case.* Two motions of one and the same point are to be represented as combined in *directions comprising an angle*.

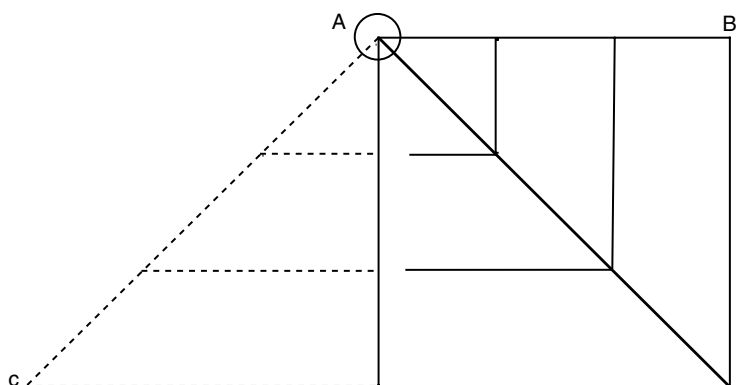


Figure 3

The two given motions are AB and AC, whose speeds and directions are expressed by these lines, but the angle comprised by these lines [is expressed] by BAC (it may, as here, be a right angle, but also any arbitrary oblique angle). Now if these two motions were to occur at the same time in the directions AB and AC, and in one and the same space, then they would still not be able to occur at the same time in these two *lines* AB and AC, but only in lines running parallel to these. It would therefore have to be assumed that one of these motions effected a change in the other (namely, directing it from the given path), if both directions were to remain the same. But this is contrary to the presupposition of the Proposition, which indicates by the word “composition” that the two given motions are to be *contained* in a third, and therefore are to be the same as the latter, and are not to produce a third, in that one *changes* the other.

By contrast, assume the motion AC as proceeding in absolute space, but, instead of the motion AB, the motion of the relative space in the opposite direction. Let the line AC be divided into three equal parts AE, EF, FC. While the body A traverses the line AE in absolute space, the relative space, together with the point E, traverses the space Ee = MA. While the body traverses the two parts together = AF, the relative space, together with the point F, describes the line Ff = NA. Finally, while the body traverses the whole line AC, the relative space, together with the point C, describes the line Cc = BA. But all of this is precisely the same as if the body A had traversed the lines Em, Fn, and CD = AM, AN, AB in [493]

these three parts of the time, and in the whole time, in which it traverses AC, the line $CD = AB$. It is thus in the last moment at the point D, and in this whole time successively at all points of the diagonal AD, which therefore expresses both the direction and speed of the composite motion.

Remark 1

Geometrical *construction* requires that one quantity be the *same* as another or that two quantities in composition be the *same* as a third, not that they produce the third as causes, which would be mechanical construction. Complete similarity and equality, insofar as it can be cognized only in intuition, is *congruence*. All geometrical construction of complete identity rests on congruence. Now this congruence of two combined motions with a third (as with the *motus compositus* itself) can never take place if these two combined motions are represented in one and the same space, for example, in relative space. Therefore, all attempts to prove the above Proposition in its three cases were always only mechanical analyses – namely, where one allows moving causes to produce a third motion by combining one given motion with another¹⁷ – but not proofs that the two motions are the same as the third, and can be represented as such a priori in pure intuition.

Remark 2

If, for example, a speed AC is called doubled, nothing else can be understood by this except that it consists of two simple and equal speeds AB and BC (see Figure 1). If, however, one explicates a doubled speed by saying that it is a motion through which a doubled space is traversed in the same time, then something is assumed here that is not obvious in itself – namely, that two equal speeds can be combined in precisely the same way as two equal spaces – and it is not clear in itself that a given speed consists of smaller speeds, and a rapidity of slownesses, in precisely the same way that a space consists of smaller spaces. For the parts of the speed are not external to one another like the parts of the space, and if the

¹⁷ This, for example, is how Newton proceeds when he derives the law of the parallelogram of velocities using his First and Second Laws of Motion in Corollary 1 to the Laws of Motion in the *Principia* (Cohen and Whitman, pp. 417–18).

former is to be considered as a quantity, then the concept of its quantity, since this is *intensive*, must be constructed in a different way from that of the *extensive* quantity of space.¹⁸ But this construction is possible in no other way than through the *mediate* composition of two equal motions, such that one is the motion of the body, and the other the motion of the relative space in the opposite direction, which, however, for precisely this reason, is entirely the same as a motion of the body in the original direction that is equal to it. For two equal speeds cannot be combined in the same body in the *same direction*, except through external moving causes, for example, a ship, which carries the body with one of these speeds, while another moving force combined immovably with the ship impresses on the body the second speed that is equal to the first. But here it must always be presupposed that the body conserves itself in *free* motion with the first speed, while the second is added – which, however, is a law of nature of moving forces that can in no way be at issue here, where the question is solely how the concept of speed as a quantity is to be *constructed*. So much, then, for the addition of speeds to one another. If, however, the subtraction of one from the other is at issue, then this can indeed easily be *thought*, as soon as the possibility of speed as a quantity through addition is granted, but this concept cannot so easily be *constructed*. For, to this end two opposite motions must be combined in one body; and how is this supposed to happen? It is impossible to think two equal motions in the same body in opposite directions immediately, that is, in relation to precisely the same space at rest. But the representation of the impossibility of these two motions in one body is not the concept of its *rest*, but rather of the *impossibility of constructing* this composition of opposite motions, which is nonetheless assumed as possible in the Proposition. This construction is possible in no other way, however, except through the combination of the motion of the body with the *motion of the space*, as was shown. Finally, with respect to the composition of two motions with directions comprising an angle, this cannot be thought in the body in reference to one and the same space either, unless we assume that one of them is effected through an *external* continually influencing *force* (for example, a vehicle carrying the body forward), while the other is conserved unchanged – or, in general, one must take as basis moving forces, and the generation of a third motion [494]

¹⁸ See A162/B203: “I call an extensive quantity that in which the representation of the parts makes possible the representation of the whole (and thus necessarily precedes the latter).” For the contrasting concept of *intensive* quantity, see A167–70/B209–12.

from two united *forces*, which is indeed the *mechanical* execution of what is contained in a concept, but not its *mathematical construction*, which should only make intuitive what the object (as quantum) *is to be*, not how it may be *produced* by nature or art by means of certain instruments and forces. – The composition of motions, in order to determine their ratio to others as quantity, must take place in accordance with the rules of congruence, which is only possible in all three cases by means of the motion of the space congruent to one of the two given motions, so that the two [together] are congruent to the composite [motion].

[495]

Remark 3

Phoronomy, not as pure doctrine of motion, but merely as pure doctrine of the quantity of motion, in which matter is thought with respect to no other property than its mere movability, therefore contains no more than this single Proposition, carried out through the above three cases, of the composition of motion – and, indeed, of the possibility of *rectilinear motions* only, not curvilinear [ones]. For since in these latter the motion is continually changed (in direction), a cause of this change must be brought forward, which cannot now be the mere space. But that one normally understood, by the term *composite motion*, only the single case where the directions comprise an angle, did no harm to physics, but rather to the principle of classification of a pure philosophical science in general. For, with respect to the former, all three cases treated in the above Proposition can be sufficiently presented in the *third alone*. For if the angle comprised by the two given motions is thought as infinitely small, then it contains the first case; but if it is represented as different from a single straight line only by an infinitely small amount, then it contains the second case; so that all three cases named by us can certainly be given in the well-known proposition of composite motion in a general formula. But one could not, in this way, learn to comprehend the doctrine of the quantity of motion a priori with respect to its parts, which also has its uses for several purposes.

If anyone is interested in connecting the above three parts of the general phoronomic Proposition with the schema of classification of all pure concepts of the understanding – namely, here that of the concept of *quantity* – then he will note that, since the concept of quantity always contains that of the composition of the homogeneous, the doctrine of the composition of motion is, at the same time, the pure doctrine of the quantity of

motion, and, indeed, in accordance with all three moments suggested by [the structure of] space: *unity* of line and direction, *plurality* of directions in one and the same line, and the *totality* of directions, as well as lines, in accordance with which the motion may occur, which contains the determination of all possible motion as a quantum, even though the quantity of motion (in a movable point) consists merely in the speed. This remark has its uses only in transcendental philosophy.

Second Chapter

Metaphysical Foundations of Dynamics

Explication 1

Matter is the *movable* insofar as it *fills* a *space*. To *fill* a space is to resist every movable that strives through its motion to penetrate into a certain space. A space that is not filled is an *empty space*.

Remark

This is now the dynamical explication of the concept of matter. It presupposes the phoronomical [explication], but adds a property relating as cause to an effect, namely, the power to resist a motion within a certain space; there could be no mention of this in the preceding science, not even when dealing with motions of one and the same point in opposite directions. This filling of space keeps a certain space free from the penetration of any other movable, when its motion is directed towards any place in this space. Now the basis for the resistance of matter exerted in all directions,^a and what this resistance is, must still be investigated. But one already sees this much from the above explication: Matter is not here considered as it resists, *when it is driven out of its place*, and thus moved itself (this case will [497] be considered later, as mechanical resistance), but rather when merely the *space* of its own extension is to be *diminished*. One uses the expression *to occupy a space*, that is, to be immediately present in all points of this space, in order to designate the *extension* of a thing in space. However, it

^a *nach allen Seiten gerichtete.*

is not determined in this concept what effect arises from this presence, or even whether there is any effect at all – whether to resist others that are striving to penetrate within; or whether it means merely a space without matter, insofar as it is a complex of several spaces, as one can say of any geometrical figure that it occupies a space by being extended; or even whether there is something in the space that compels another movable to penetrate deeper into it (by attracting others) – because, I say, all this is undetermined by the concept of occupying a space, *filling a space* is a more specific determination of the concept of *occupying* a space.

Proposition 1

Matter fills a space, not through its mere *existence*, but through a *particular moving force*.

Proof

Penetration into a space (in the initial moment this is called a striving to penetrate) is a motion. Resistance to motion is the cause of its diminution, or even of the change of this motion into rest. Now nothing can be combined with a motion, which diminishes it or destroys it, except another motion of precisely the same movable in the opposite direction (Phoron. Prop.). Therefore, the resistance that a matter offers in the space that it fills to every penetration by other matters is a cause of the motion of the latter in the opposite direction. But the cause of a motion is called a moving force. Thus matter fills its space through a moving force, and not through its mere existence.

Remark

Lambert and others called the property of matter by which it fills a space *solidity* (a rather ambiguous expression), and claim that one must assume this in every thing *that exists* (substance), at least in the outer sensible world. According to their ideas^b the presence of something *real* in space¹⁹ must already, through its concept, and thus in accordance with the [498]

^b *Begriffen*.

¹⁹ For the concept of “the *real* in space” compare A173/B215.

principle of noncontradiction, imply this resistance, and bring it about that nothing else can be simultaneously in the space where such a thing is present. But the principle of noncontradiction does not repel a matter advancing to penetrate into a space where another is found. Only when I ascribe to that which occupies a space a force to repel every external movable that approaches, do I understand how it contains a contradiction for yet another thing of the same kind to penetrate into the space occupied by a thing. Here the mathematician has assumed something, as a first datum for constructing the concept of a matter, which is itself incapable of further construction. Now he can indeed begin his construction of a concept from any chosen datum, without engaging in the explication of this datum in turn. But he is not therefore permitted to declare this to be something entirely incapable of any mathematical construction,^c so as thereby to obstruct us from going back to first principles in natural science.

Explication 2

Attractive force is that moving force by which a matter can be the cause of the approach of others to it (or, what is the same, by which it resists the removal of others from it).

Repulsive force is that by which a matter can be the cause of others removing themselves from it (or, what is the same, by which it resists the approach of others to it). The latter force will also sometimes be called *driving force*, the former *drawing force*.

Note

Only these two moving forces of matter can be thought. For all motion that one matter can impress on another, since in this regard each of them is considered only as a point, must always be viewed as imparted in the straight line between the two points. But in this straight line there are only two possible motions: the one through which the two points *remove* themselves from one another, the second through which they *approach* one another. But the force causing the first motion is called *repulsive force*, whereas the second is called *attractive force*. Therefore, only these two

[499]

^c jenes für etwas aller mathematischen Construction ganz Unfähiges zu erklären.

kinds of forces can be thought, as forces to which all moving forces in material nature must be reduced.

Proposition 2

Matter fills its space through the repulsive forces of all of its parts, that is, through an expansive force of its own, having a determinate degree, such that smaller or larger degrees can be thought to infinity.

Proof

Matter fills a space only through moving force (Prop. 1), a force resisting the penetration (that is, the approach) of others. Now this is a repulsive force (Explication 2). Therefore, matter fills its space only through repulsive forces, and, indeed, through repulsive forces of all of its parts. For otherwise a part of its space (contrary to the presupposition) would not be filled, but only enclosed. But *the force of something extended in virtue of the repulsion of all of its parts* is an *expansive force*. So matter fills its space only through an expansive force of its own, *which was the first* [thing to be shown]. Now, beyond any given force a greater force must be thinkable, for that force beyond which no greater is possible would be one whereby an infinite space would be traversed in a finite time (which is impossible). Further, below any given force a smaller force must be thinkable (for the smallest force would be one whereby its infinite addition to itself throughout a given time could generate no finite speed, which, however, means the absence of all moving force). Thus, below any given degree of a moving force a smaller must always be capable of being given, *which is the second* [thing to be shown]. Consequently, the expansive force by which every matter fills its space has a degree, which is never the greatest or the smallest, but is such that beyond it both greater and smaller degrees can be found to infinity.

Note 1

[500]

The expansive force of a matter is also called *elasticity*. Now, since it is the basis on which the filling of space rests, as an essential property of all matter, this elasticity must therefore be called *original*, because it can be

derived from no other property of matter. All matter is therefore originally elastic.

Note 2

Beyond every expanding force a greater moving force can be found. But the latter can also act contrary to the former, whereby it would then decrease the space that the former strives to enlarge, in which case the latter would be called *compressing* force. Therefore, for every matter a compressing force must also be discoverable, which can drive it from the space it fills into a decreased space.

Explication 3

A matter *penetrates* another in its motion, when it completely destroys the space of the latter's extension through compression.

Remark

When, in the barrel of an air pump filled with air, the piston is driven closer and closer to the bottom, the air-matter is compressed. If this compression could now be driven so far that the piston completely touched the bottom (without the least amount of air escaping), then the air-matter would be penetrated. For the matters enclosing the air would leave no remaining space for it, and it would thus be found between the piston and the bottom without occupying a space. This penetrability of matter through external compressing forces, if someone wished to assume or even to think such a thing, could be called *mechanical* penetration. I have reason thus to distinguish this penetrability of matter from another kind, whose concept is perhaps just as impossible as the first, but of which I may yet have occasion to say something later on.²⁰

[501]

Proposition 3

Matter can be *compressed* to infinity, but can *never* be *penetrated* by a matter, no matter how great the compressing force of the latter may be.

²⁰ See, in particular, the discussion of *chemical* penetration under No. 4 of the General Remark to Dynamics [530–32].

Proof

An original force, with which a matter strives to extend itself on all sides beyond a given space that it occupies, must be greater when enclosed in a smaller space, and infinite when compressed into an infinitely small space. Now, for a given expanding force of matter, a greater compressing force can be found, which forces the former into a smaller space, and so on to infinity, which was the first [thing to be shown]. But a compression of matter into an infinitely small space, and thus an infinite compressing force, would be required for its penetration, and this is impossible. Therefore, a matter cannot be penetrated by any other through compression, which is the second [thing to be shown].

Remark

In this proof I have assumed from the very beginning that an expanding force must counteract all the more strongly, the more it is driven into a smaller space. But this would not in fact hold for every kind of merely derivative elastic forces. However, it can be postulated in matter, insofar as essential elasticity belongs to it, as matter in general filling a space. For expansive force, exerted from every point, and in every direction, actually constitutes this concept. But the same quantum of extending forces, when brought into a smaller space, must repel all the more strongly at every point, the smaller the space in which this quantum diffuses its activity.

Explication 4

I call the *impenetrability* of matter that rests on resistance increasing in proportion to the degree of compression *relative impenetrability*. But that resting on the *presupposition* that matter as such is capable of no [502] compression at all is called *absolute impenetrability*. The *filling of space* with absolute impenetrability can be called *mathematical filling of space*, whereas that with mere relative impenetrability can be called *dynamical filling of space*.²¹

²¹ Compare the General Remark to Dynamics for the contrast between the “mathematical” and “dynamical” natural philosophies [532–35].

Remark 1

According to the purely mathematical concept of impenetrability (which presupposes no moving force as originally belonging to matter), matter is not capable of compression except insofar as it contains empty spaces within itself. Hence matter as matter resists all penetration utterly^d and with absolute necessity. However, according to our discussion of this property, impenetrability rests on a physical basis. For expanding force first makes matter itself possible, as an extended thing filling its space. But this force has a degree that can be overpowered, and thus the space of its extension can be diminished, that is, penetrated up to a certain amount by a given compressing force, but only in such a way that complete penetration is impossible, because this would require an infinite compressing force; *therefore the filling of space must be viewed only as relative impenetrability.*

Remark 2

Absolute impenetrability is in fact nothing more nor less than an occult quality.^e For one asks what the cause is for the inability of matters to penetrate one another in their motion, and one receives the answer: because they are impenetrable. The appeal to repulsive force is not subject to this reproach. For, although this force cannot be further explicated in regard to its possibility, and therefore must count as a fundamental force, it does yield a concept of an acting cause, together with its laws, whereby the action, namely, the resistance in the filled space, can be estimated in regard to its degrees.

Explication 5

Material substance is that in space which is movable in itself, that is, in isolation from everything else existing external to it in space. The motion [503] of a part of matter, whereby it ceases to be a part, is *separation*. The separation of the parts of a matter is *physical division*.

Remark

The concept of a substance means the ultimate subject of existence, that is, that which does not itself belong in turn to the existence of another

^d *schlechterdings.* ^e qualitas occulta.

merely as a predicate. Now matter is the subject of everything that may be counted in space as belonging to the existence of things. For, aside from matter, no other subject would be thinkable except space itself, which, however, is a concept that contains nothing existent at all, but merely the necessary conditions for the external relations of possible objects of the outer senses. Thus matter, as the movable in space, is the substance therein. But all parts of matter must likewise be called substances, and thus themselves matter in turn, insofar as one can say of them that they are themselves subjects, and not merely predicates of other matters. They are themselves subjects, however, if they are movable in themselves, and thus exist in space outside their connection with other neighboring parts. Therefore, the movability belonging to matter, or any part of it, is at the same time a proof that this movable thing, and any movable part thereof, is substance.

Proposition 4

Matter is *divisible to infinity*, and, in fact, into parts such that each is matter in turn.

Proof

Matter is impenetrable, through its original expansive force (Prop. 3²²). But this is only a consequence of the repulsive forces of each point in a space filled with matter. Now the space filled by matter is mathematically divisible to infinity, that is, its parts can be distinguished to infinity, although they cannot be moved, and thus cannot be divided (according to geometrical proofs). But in a space filled with matter, every part of it contains repulsive force, so as to counteract all the rest in all directions, and thus to repel them and to be repelled by them, that is, to be moved a distance from them. Hence, every part of a space filled with matter is [504] in itself movable, and thus separable from the rest as material substance through physical division. Therefore, the possible physical division of the substance that fills space extends as far as the mathematical divisibility of the space filled by matter. But this mathematical divisibility extends to infinity, and thus so does the physical [divisibility] as well. That is, all

²² It appears that "Prop. 2" is meant.

matter is divisible to infinity, and, in fact, into parts such that each is itself material substance in turn.

Remark 1

The proof of the infinite divisibility of space has not yet come close to proving the infinite divisibility of matter, if it has not previously been shown that there is material substance in every part of space, that is, that parts movable in themselves are to be found there. For suppose that a *monadist* wished to assume that matter consisted of physical points, each of which (for precisely this reason) had no movable parts, but nonetheless filled a space through mere repulsive force. Then he could grant that space would be divided, but not the substance that acts in space – that the sphere of activity of this substance [would be divided] by the division of space, but not the acting movable subject itself.²³ Thus he would assemble matter out of physically indivisible parts, and yet allow them to occupy a space in a *dynamical fashion*.

But this way out is completely taken away from the monadist by the above proof. For it is thereby clear that there can be no point in a filled space that does not exert repulsion in all directions, and is itself repelled, and thus would be movable in itself, as a reacting subject external to every other repelling point. Hence the hypothesis of a point that would fill a space through mere driving force, and not by means of other equally repelling forces, is completely impossible. In order to make this intuitive, and hence also the proof of the preceding Proposition, let us assume that *A* is the place of a monad in space, and *ab* is the diameter of the sphere of its repulsive force, so that *aA* is the radius of this sphere:

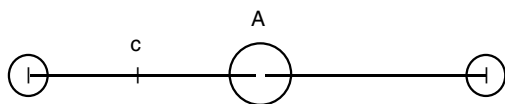


Figure 4

Then between *a*, where the penetration of an external monad into the space occupied by this sphere is resisted, and the center *A*, it is possible

²³ In the *Physical Monadology* of 1756 Kant argues that a monad can fill a space in virtue of the “sphere of activity [*sphaera activitatis*]” of its repulsive force without detriment to the absolute simplicity and indivisibility of the substantial monad itself: Propositions IV–VII (Ak 1:479–82).

to specify a point *c* (according to the infinite divisibility of space). But if *A* resists that which strives to penetrate into *a*, then *c* must also resist the two points *A* and *a*. For, if this were not so, they would approach one another without hindrance, and thus *A* and *a* would meet at the point *c*, that is, the space would be penetrated. Therefore, there must be something at *c* that resists the penetration of *A* and *a*, and thus repels the monad *A*, the same as it is also repelled by *A*. But since *repelling* is a [kind of] moving, *c* is something movable in space, and thus matter, and the space between *A* and *a* could not be filled through the sphere of activity of a single monad, nor could the space between *c* and *A*, and so on to infinity. [505]

When mathematicians represent the repulsive forces of the parts of elastic matters as increasing or decreasing, in accordance with a certain proportion of their distances from one another, at greater or lesser compression of these parts (for example, that the smallest parts of the air repel one another in inverse ratio to their distances from one another, because the elasticity of these parts stands in inverse ratio to the spaces in which they are compressed), then one completely misses their meaning, and misinterprets their language, if one ascribes that which necessarily belongs to the procedure of constructing a concept to the concept in the object itself. For by the latter [procedure], any contact can be represented as an infinitely small distance – which must also necessarily be so in those cases where a greater or smaller space is to be represented as completely filled by one and the same quantity of matter, that is, one and the same quantum of repulsive forces. So even in the case of something divisible to infinity, no actual distance of the parts may therefore be assumed – they always constitute a continuum, no matter how enlarged is the space, even though the possibility of such an enlargement can only be made intuitive under the idea of an infinitely small distance.

Remark 2

To be sure, mathematics in its internal use can be entirely indifferent with regard to the chicanery of a misguided metaphysics, and can persist in the secure possession of its evident claims as to the *infinite divisibility of space*, whatever objections may be put in its way by a sophistry splitting hairs on mere concepts. However, in the application of its propositions governing space to the substance that fills it, mathematics must

nonetheless accede to an examination in accordance with mere concepts, and thus to metaphysics. The above Proposition is already a proof of this. For it does not necessarily follow that matter is physically divisible to infinity, even if it is so from a mathematical point of view, even if every part of space is a space in turn, and thus always contains [more] parts external to one another. For so far it cannot be proved that in each of the possible parts of this *filled* space there is also *substance*, which therefore also exists in separation from all else as movable in itself. Thus something without which this proof could not find secure application to natural science was until now still missing in the mathematical proof, and this deficiency is [506] remedied in the above Proposition. Now, however, when it comes to the remaining metaphysical attacks on what will henceforth be the *physical Proposition* of the infinite divisibility of matter, the mathematician must leave them entirely to the philosopher, who in any case ventures, by means of these objections, into a labyrinth, from which it becomes difficult for him to extricate himself, even in those questions immediately pertaining to him. He therefore has quite enough to do for himself, without the mathematician being permitted to involve himself in this business. For if matter is divisible to infinity then (concludes the dogmatic metaphysician) *it consists of an infinite aggregate of parts*; for a whole must already contain in advance all of the parts in their entirety, into which it can be divided. And this last proposition is undoubtedly certain for every whole *as thing in itself*. But one cannot admit that matter, or even space, *consists of infinitely many parts* (because it is a contradiction to think an infinite aggregate, whose concept already implies that it can never be represented as completed, as entirely completed). One would therefore have to conclude either, in spite of the geometer, that *space is not divisible to infinity*, or, to the annoyance of the metaphysician, that *space is not a property of a thing in itself*, and thus that matter is not a thing in itself, but merely an appearance of our outer senses in general, just as space is the essential form thereof.

But here the philosopher is caught between the horns of a dangerous dilemma. To deny the first proposition, that space is divisible to infinity, is an empty undertaking; for nothing can be argued away from mathematics by sophistical hair-splitting.^f But viewing matter as a thing in itself, and thus space as a property of the thing in itself, amounts to the denial of this

^f *dem Mathematik läßt sich nichts wegvernünfteln.*

proposition. The philosopher therefore finds himself forced to deviate from this last proposition,²⁴ however common and congenial to the common understanding it may be. But he does this, of course, only provided that, after making matter and space into mere appearances (and thus the latter into the form of our outer sensible intuition, so that both [are made] not into things in themselves, but only into subjective modes of representation of objects unknown to us in themselves), he is thereby helped out of that difficulty due to the *infinite divisibility* of matter, whereby it still does *not* consist of *infinitely many parts*. Now this latter can perfectly well be thought through reason, even though it cannot be made intuitive and constructed. For what is only actual by being given in the representation also has no *more* given of it than what is met with in the representation – no more, that is, than the progress of representations reaches. Therefore, one can only say of appearances, whose division proceeds to infinity, that there are just so many parts in the appearance as we may provide, that is, so far as we may divide. For the parts, as belonging to the existence [507] of an appearance, exist only in thought, namely, in the division itself. Now, the division does of course proceed to infinity, but it is still never given as infinite. Thus it does not follow, from the fact that its division proceeds to infinity, that the divisible contains an infinite aggregate of parts *in itself*, and outside of our representation. For it is not the thing, but only this representation of it, whose division, although it can indeed be continued to infinity, and there is also a ground for this in the object (which is unknown in itself), can nonetheless never be completed, and thus be completely given; and this also proves no actual infinite aggregate in the object (which would be an explicit contradiction).²⁵ A great man,²⁶ who has contributed perhaps more than anyone else to preserving the reputation of mathematics in Germany, has frequently rejected the presumptuous metaphysical claims to overturn the theorems of geometry concerning the infinite divisibility of space by the well-founded reminder *that space belongs only to the appearance of outer things*. But he has not been understood. This proposition was taken to be asserting that space appears to us, though it is otherwise a thing, or relation of things, in itself, but

²⁴ That is, the proposition that matter is a thing in itself.

²⁵ See the discussion of the Second Antinomy in the first *Critique*: A434–43/B462–71, A523–27/B551–55.

²⁶ Various possibilities have been suggested here, the most plausible of which are Leibniz, Euler, and Lambert. In view of the rest of the passage, Leibniz seems to be the most probable.

that the mathematician considers it only as it appears. Instead, it should have been understood as saying that space is in no way a property that attaches in itself to any thing at all outside our senses. It is, rather, only the subjective form of our sensibility, under which objects of the outer senses, with whose constitution in itself we are not acquainted, appear to us, and we then call this appearance matter. Through this misunderstanding one went on thinking of space as a property also attaching to things outside our faculty of representation, but such that the mathematician thinks it only in accordance with common concepts,^g that is, confusedly (for it is thus that one commonly explicates appearance). And one thus attributed the mathematical theorem of the infinite divisibility of matter, a proposition presupposing the highest [degree of] clarity in the concept of space, to a confused representation of space taken as basis by the geometer – whereby the metaphysician was then free to compose space out of points, and matter out of simple parts, and thus (in his opinion) to bring clarity into this concept. The ground for this aberration lies in a poorly understood *monadology*, [a theory] which has nothing at all to do with the explanation of natural appearances, but is rather an intrinsically correct *platonic* concept of the world devised by *Leibniz*, insofar as it is considered, not at all as object of the senses, but as thing in itself, and is merely an object of the understanding, which, however, does indeed underlie the appearances of the senses. Now the *composite of things in themselves* must certainly consist of the simple, for the parts must here be given prior to all composition. But the *composite in the appearance* does not consist of

[508] the simple, because in the appearance, which can never be given otherwise than as composed (extended), the parts can only be given through division, and thus not prior to the composite, but only in it. Therefore, *Leibniz's* idea,^h so far as I comprehend it, was not to explicate space through the order of simple beings next to one another, but was rather to set this order alongside space as corresponding to it, but as belonging to a merely intelligible world (unknown to us). Thus he asserts nothing but what has been shown elsewhere: namely, that space, together with the matter of which it is the form, does not contain the world of things in themselves, but only their appearance, and is itself only the form of our outer sensible intuition.

^g *gemeinen Begriffen*.

^h *Meinung*.

Proposition 5

The possibility of matter requires an *attractive force* as the second essential fundamental force of matter.

Proof

Impenetrability, as the fundamental property of matter, whereby it first manifests itself to our outer senses, as something real in space, is nothing but the expansive power of matter (Proposition 2). Now an essential moving force, whereby the parts of matter flee from one another, cannot, *in the first place*, be limited by itself, for matter is thereby striving instead continuously to enlarge the space that it fills; *in the second place*, [such a force] can also not be determined by space alone to a certain limit of extension, for the latter, although it can certainly contain the ground for the expansive force becoming weaker in inverse proportion to the increase of volume of an expanding matter, can never contain the ground for this force ceasing anywhere, because smaller degrees are possible to infinity for any moving force. Hence matter, by its repulsive force (containing the ground of impenetrability), would, [through itself] alone and if no other moving force counteracted it, be confined within no limit of extension; that is, it would disperse itself to infinity, and no specified quantity of matter would be found in any specified space. Therefore, with merely repulsive forces of matter, all spaces would be empty, and thus, properly speaking, no matter would exist at all. So all matter requires for its existence forces that are opposed to the expansive forces, that is, compressing [509] forces. But these, in turn, cannot originally be sought in the contrary strivingⁱ of another matter, for this latter itself requires a compressive force in order to be matter. Hence there must somewhere be assumed an original force of matter acting in the opposite direction to the repulsive force, and thus to produce approach, that is, an attractive force. Yet since this attractive force belongs to the possibility of a matter as matter in general, and thus precedes all differences of matter, it may not be ascribed merely to a particular species of matter, but must rather be ascribed to all matter originally and as such. Therefore, an original attraction is attributed to all matter, as a fundamental force belonging to its essence.

ⁱ *Entgegenstrebung*.

Remark

In this transition from one property of matter to another, specifically different from it, and belonging equally to the concept of matter, *even though not contained in it*, the procedure of our understanding must be considered more closely. If attractive force is originally required even for the possibility of matter, why do we not use it, just as much as impenetrability, as the first distinguishing mark of a matter? Why is the latter immediately given with the concept of a matter, whereas the former is not thought in the concept, but only adjoined to it through inferences? That our senses do not allow us to perceive this attraction so immediately as the repulsion and resistance of impenetrability cannot yet provide a sufficient answer to the difficulty. For even if we had such a capacity, it is still easy to see that our understanding would nonetheless choose the filling of space in order to designate substance in space, that is, matter, and how precisely this *filling*, or, as one otherwise calls it, *solidity*, is then posited to be characteristic of matter, as a thing different from space. Attraction, even if we sensed it equally well, would still never disclose to us a matter of determinate *volume* and *figure*, but only the striving of our organ to approach a point outside us (the center of the attracting body). For the attractive force of all parts of the earth can affect us no more, and in no other way, than as if it were wholly united in the earth's center, and this alone influenced our sense, and the same holds for the attraction of a mountain, or any stone, etc.²⁷ But we thereby obtain no determinate concept of any object in space, since neither figure, nor quantity, nor even the place where it

[510] would be found can strike our senses. (The mere direction of attraction would be perceivable, as in the case of weight: the attracting point would be unknown, and I do not even see how it could be ascertained through inferences, without perception of matter insofar as it fills space). It is therefore clear that the first application of our concepts of *quantity* to matter, through which it first becomes possible for us to transform our outer perceptions into the empirical concept of a matter, as object in general, is grounded only on that property whereby it fills a space – which, by means of the sense of feeling,^j provides us with the quantity and figure

^j Gefühl.

²⁷ Proposition 8 of Book III of Newton's *Principia* states that the gravitational attraction of a sphere whose mass is distributed symmetrically about its center acts as if all the mass were concentrated there (Cohen and Whitman, p. 811). This does not hold, however, for arbitrary mass distributions.

of something extended, and thus with the concept of a determinate object in space, which forms the basis of everything else one can say about this thing. Precisely this circumstance is undoubtedly the reason, despite the clearest proofs from elsewhere that attraction must belong to the fundamental forces of matter, just as much as repulsion, that one nevertheless struggles so much against the former, and will admit no other moving forces at all except those through impact and pressure (both mediated by impenetrability). For that whereby space is filled is substance, one says, and this is also perfectly correct. But this substance discloses its existence to us in no other way than through that sense whereby we perceive its impenetrability, namely, feeling, and thus only in relation to contact, whose onset (in the approach of one matter to another) is called impact, and whose persistence is called pressure. It therefore seems as if every immediate action of one matter on the other could never be anything but pressure or impact, the only two influences we can sense immediately. Attraction, on the other hand, can give us in itself either no sensation at all, or at least no determinate object of sensation, and it is therefore so difficult for us to understand as a fundamental force.

Proposition 6

No matter is possible through mere attractive force without repulsion.

Proof

Attractive force is that moving force of matter whereby it impels another to approach it; consequently, if it is found between all parts of matter, matter thereby strives to diminish the distance of its parts from one another, and thus the space that they occupy together. But nothing can hinder [511] the action of a moving force except another moving force opposed to it, and that which opposes attraction is repulsive force. Hence, without repulsive forces, through mere convergence,^k all parts of matter would approach one another unhindered, and would diminish the space that they occupy. But since, in the case assumed, there is no distance of the parts at which a greater approach due to attraction would be made impossible by a repulsive force, they would move towards one another so far, until no

^k *Annäherung*.

distance at all would be found between them; that is, they would coalesce into a mathematical point, and space would be empty, and thus without any matter. Therefore, matter is impossible through mere attractive forces without repulsive forces.

Note

A property on which the inner possibility of a thing rests, as a condition, is an essential element thereof. Hence repulsive force belongs to the essence of matter just as much as attractive force, and neither can be separated from the other in the concept of matter.

Remark

Since only two moving forces can be thought everywhere in space,¹ repulsion and attraction, it was previously necessary, in order to prove a priori the uniting of the two in the concept of a matter in general, that each be considered on its own, so as to see what either in isolation could achieve for the presentation of a matter. It is now manifest that, whether one takes neither as basis, or assumes merely one of them, space would always remain empty, and no matter would be found therein.

Explication 6

Contact in the physical sense is the immediate action and reaction of *impenetrability*. The action of one matter on another in the absence of contact is *action at a distance* (actio in distans). This action at a distance, which is possible even without the mediation of matter lying in between, [512] is called immediate action at a distance, or the *action* of matters on one another *through empty space*.

Remark

Contact in the mathematical sense is the common boundary of two spaces, which is therefore within neither the one nor the other space. Thus two straight lines cannot be in contact with one another; rather, if they have a

¹ Weil überall nur zwei bewegende Kräfte im Raum gedacht werden können.

point in common, it belongs as much to one of these lines as to the other when they are produced,^m that is, they intersect. But a circle and a straight line, or two circles, are in contact at a point, surfaces at a line, and bodies at surfaces. Mathematical contact is the basis for physical contact, but does not yet constitute the latter by itself, since for the one to arise from the other a dynamical relation must also be added in thought – and, indeed, not of attractive, but of repulsive forces, that is, of impenetrability. Physical contact is the interaction of repulsive forces at the common boundary of two matters.

Proposition 7

The *attraction essential to all matter* is an immediate action of matter on other matter through empty space.

Proof

The original attractive force contains the very ground of the possibility of matter, as that thing which fills a space to a determinate degree, and so contains even [the ground] of the possibility of a physical contact thereof. It must therefore precede the latter, and its action must thus be independent of the condition of contact. But the action of a moving force that is independent of all contact is also independent of the filling of space between the moving and the moved [matters]; that is, it must also take place without the space between the two being filled, and thus as action through empty space. Hence the original attraction essential to all matter is an immediate action of matter on other matter through empty space.

Remark 1

[513]

That the possibility of the fundamental forces should be made conceivable is a completely impossible demand; for they are called fundamental forces precisely because they cannot be derived from any other, that is, they can in no way be conceived. But the original attractive force is in no way *more inconceivable* than the original repulsion. It simply does not present itself so immediately to the senses as impenetrability, so as to furnish us with

^m *fortgezogen*.

concepts of determinate objects in space. Thus, because it is not felt, but is only to be inferred,ⁿ it has so far the appearance of a derived force, exactly as if it were only a hidden play of moving forces through repulsion. On closer consideration we see that it can in no way be further derived from anywhere else, least of all from the moving force of matters through their impenetrability, since its action is precisely the reverse of the latter. The most common objection to immediate action at a distance is that a matter cannot act immediately *where it is not*. When the earth immediately impels the moon to approach it, the earth acts on a thing that is many thousands of miles away from it, and yet immediately; the space between it and the moon may well be viewed as completely empty. For even though matter may lie between the two bodies, it still contributes nothing to this attraction. It therefore acts immediately at a place where it is not, which is apparently contradictory. In truth, however, it is so far from being contradictory that one may rather say that every thing in space acts on another only at a place where the acting thing is not. For if it should act at the same place where it itself is, then the thing on which it acts would not be *outside it* at all; for this *outsideness* means presence at a place where the other is not. If earth and moon were to be in contact with one another, the point of contact would still be a place where neither the earth nor the moon is, for the two are distanced from one another by the sum of their radii. Moreover, no part of either the earth or the moon would be found at the point of contact, for this point lies at the boundary of the two filled spaces, which constitutes no part of either the one or the other. Hence to say that matters cannot act immediately on one another at a distance, would amount to saying that they cannot act immediately on one another except through the forces of impenetrability. But this would be as much as to say that repulsive forces are the only ones whereby matters can be active, or that they are at least the necessary conditions under which alone matters can act on one another, which would declare attractive force to be either completely impossible or always dependent on the action of repulsive forces. But these are both groundless assertions. The confusion of mathematical contact

[514] of spaces and physical contact through repulsive forces constitutes the ground of misunderstanding here. To attract one another immediately in the absence of contact means to approach one another in accordance with an invariable law, without a force of repulsion containing the condition

ⁿ *Weil sie also nicht gefühlt, sondern nur geschlossen werden will.*

for this. And this must be just as thinkable as an immediate repulsion of one another, that is, to flee from one another in accordance with an invariable law, without the force of attraction having any part therein. For the two moving forces are of completely different kinds, and there is not the slightest ground for making one of them dependent on the other, and contesting its possibility unmediated by the other.

Remark 2

No motion at all can arise from attraction in contact; for contact is interaction of impenetrability, which therefore prevents all motion. Hence, some sort of immediate attraction must be found in the absence of contact, and thus at a distance. For otherwise even the forces of pressure and impact, which are supposed to bring about the striving to approach by acting in the opposite direction to that of the repulsive force of matter, would have no cause, or at least none lying originally in the nature of matter. We may call that attraction which takes place without mediation of the repulsive forces *true* attraction, whereas that which takes place merely in that way is *apparent* attraction. For, properly speaking, the body which another is striving to approach, merely because the latter has been driven towards it from elsewhere by impact, exerts no attractive force at all on this body. But even these apparent attractions must in the end have a true one as their ground. For matter whose pressure or impact is supposed to serve instead of attraction would not even be matter without attractive forces (Proposition 5), and so the mode of explaining all phenomena of approach by *merely apparent* attraction revolves in a circle. It is commonly supposed that Newton did not at all find it necessary for his system to assume an immediate attraction of matter, but, with the most rigorous abstinence of pure mathematics, allowed the physicists full freedom to explain the possibility of attraction as they might see fit, without mixing his propositions with their play of hypotheses. But how could he ground the proposition that the universal attraction of bodies, which they exert at equal distances around them, is proportional to the quantity of their matter, if he did not assume that all matter, merely as matter, therefore, and through its essential property, exerts this moving force? For although between two bodies, when one attracts the other, whether their matter be similar or not, the mutual approach (in accordance with the law of equality of interaction) must always occur in inverse ratio to the quantity of matter, this [515]

law still constitutes only a principle of mechanics, but not of dynamics. That is, it is a law of the *motions* that follow from attracting forces, not of the proportion of the *attractive forces* themselves, and it holds for all moving forces in general.²⁸ Thus, if a magnet is at one time attracted by another equal magnet, and at another by the same magnet enclosed in a wooden box of double the weight, the latter will impart more relative motion to the former in the second case than the first, even though the wood, which increases the quantity of matter of this second magnet, adds nothing at all to its attractive force, and manifests^o no magnetic attraction of the box. Newton says (Cor. 2, Prop. 6, Book III, *Principia*): “if the aether or any other body were without weight, it could, since it differs from every other matter only in its form, be transformed successively by gradual change of this form into a matter of the same kind as those which on earth have the most weight; and so the latter, conversely, by gradual change of their form, could lose all their weight, which is contrary to experience, etc.”²⁹ Thus he did not himself exclude the aether (much less other matters) from the law of attraction. So what other kind of matter could he then have left, by whose impact the approach of bodies to one another might be viewed as mere apparent attraction? Thus, one cannot adduce this great founder of the theory of attraction as one’s predecessor, if one takes the liberty of substituting an apparent attraction for the true attraction he did assert, and assumes the *necessity* of an impulsion through *impact* to explain the phenomenon of approach. He rightly abstracted from all hypotheses purporting to answer the question as to the cause of the

^o beweiset.

²⁸ In modern terms, the distinction here is between *inertial mass*, which governs the interaction of any forces at all in accordance with Newton’s Second and Third Laws of Motion, and specifically *gravitational mass*, which plays a crucial role (both “active” and “passive”) in the law of universal gravitation. Thus, the equality subsisting between inertial mass and (passive) gravitational mass implies that the acceleration of a gravitationally attracted body is independent of its particular constitution (all bodies fall the same in a gravitational field). This is certainly not true of electric and magnetic forces, for example, where the corresponding acceleration depends explicitly on the mass–charge ratio of the body in question.

²⁹ Kant’s rendering deviates from Newton’s text, which, in the translation of Cohen and Whitman, reads rather as follows (p. 809): “If the aether or any other body whatever either were entirely devoid of gravity or gravitated less in proportion to its quantity of matter, then, since (according to the opinion of Aristotle, Descartes, and others) it does not differ from other bodies except in the form of its matter, it could by a change of its form be transmuted by degrees into a body of the same condition as those that gravitate the most in proportion to the quantity of their matter; and, on the other hand, the heaviest bodies, through taking on by degrees the form of the other body, could by degrees lose their gravity. And accordingly the weights would depend upon the forms of bodies and could be altered with the forms, contrary to what has been proved in corol. 1.”

universal attraction of matter, for this question is physical or metaphysical, but not mathematical. And, even though he says in the advertisement to the second edition of his *Optics*, “to show that I do not take *gravity* for an *essential* property of bodies, I have added one question concerning its cause,”³⁰ it is clear that the offense taken by his contemporaries, and perhaps even by Newton himself, at the concept of an original attraction set him at variance with himself. For he could by no means say that the attractive forces of two planets, those of Jupiter and Saturn for example, manifested at equal distances of their satellites (whose mass is unknown), are proportional to the quantity of matter of these heavenly bodies,³¹ if he did not assume that they attracted other matter merely as matter, and thus according to a universal property of matter.

Explication 7

[516]

I call a moving force whereby matters can act immediately on one another only at the common surface of contact, a *surface force*. But that whereby a matter can act immediately on the parts of others, even beyond the surface of contact, I call a *penetrating force*.

Note

The repulsive force whereby matter fills a space is a mere surface force, for the parts in contact mutually limit their spaces of action. Repulsive force cannot move a part at a distance without the mediation of those lying in between, and an immediate action, passing straight through the latter, of one matter on another by expansive forces, is impossible. By contrast, no

³⁰ Kant quotes from the second Latin edition: ne quis gravitatem inter essentielles corporum proprietates me habere existimet, quaestionem unam de eius causa investiganda subieci. Newton’s reference is to Query 21, where he famously speculates that a universal “Aetherial Medium” growing denser at greater distances from the heavenly bodies might “thereby cause the gravity of those great Bodies towards one another, and of their parts towards the Bodies: every Body endeavoring to go from the denser parts of the Medium towards the rarer” (*Opticks*, 4th edn, 1730: p. 350).

³¹ In Corollaries 1 and 2 of Proposition 8 of Book III of the *Principia* Newton determines first the (relative) weights and then the (relative) masses of Jupiter and Saturn from the distances and periodic times of their satellites. This argument depends on the previous Proposition 7, which shows that the force of gravity exerted by a body is proportional to the mass of that body (Cohen and Whitman, pp. 810–11). In the terminology of note 28, then, what is here crucial is the proportionality of gravitational force to (active) gravitational mass and the equality of the latter to inertial mass.

intervening matter sets limits to the action of an attractive force, whereby matter occupies a space *without filling it*, so that it thereby acts on other distant matter *through empty space*. Now the original attraction, which makes matter itself possible, must be thought in this way, and it is therefore a penetrating force, and for this reason alone is always proportional to the quantity of matter.

Proposition 8

The original attractive force, on which the very possibility of matter as such rests, extends immediately to infinity throughout the universe, from every part of matter to every other part.

Proof

Because the original attractive force belongs to the essence of matter, it also pertains to every part of matter to act immediately at a distance as well. But suppose there were a distance beyond which it did not extend. Then [517] this *limiting* of the sphere of its activity would rest either on the *matter* lying within this sphere, or merely on the magnitude of the *space* in which it diffuses this influence. The first [case] does not hold; for this attraction is a penetrating force and acts *immediately* at a distance through that space, as an empty space, regardless of any matter lying in between. The second [case] likewise does not hold; for, since every attraction is a moving force having a degree, below which ever smaller degrees can always be thought to infinity, a greater distance would indeed be a reason for the degree of attraction to diminish in inverse ratio, in accordance with the measure of the diffusion of this force, but never for it to cease altogether. Thus, since there is nothing that has anywhere limited the sphere of activity of the original attraction of every part of matter, it extends beyond all specified limits to every other matter, and thus throughout the universe to infinity.

Note 1

From this original attractive force, as a penetrating force exerted by all matter, and hence in proportion to its quantity, and extending its action to all matter at all possible distances, it should now be possible, in combination with the force counteracting it, namely, repulsive force, to derive the

limitation of the latter, and thus the possibility of a space filled to a determinate degree. And thus the dynamical concept of matter, as that of the movable filling its space (to a determinate degree), would be constructed. But for this one needs a law of the ratio of both original attraction and repulsion at various distances of matter and its parts from one another, which, since it now rests simply on the difference in direction of these two forces (where a point is driven either to approach others or to move away from them), and on the magnitude of the space into which each of these forces diffuses at various distances, is a purely mathematical task, which no longer belongs to metaphysics – nor is metaphysics responsible if the attempt to construct the concept of matter in this way should perhaps not succeed. For it is responsible only for the correctness of the elements of the construction granted to our rational cognition, not for [518] the insufficiency and limits of our reason in carrying it out.

Note 2

Since every given matter must fill its space with a determinate degree of repulsive force, in order to constitute a determinate material thing, only an original attraction in conflict with the original repulsion can make possible a determinate degree of the filling of space, and thus matter. Now it may be that the former flows from the individual attraction of the parts of the compressed matter among one another, or from the uniting of this attraction with that of all matter in the universe.^p

The original attraction is proportional to the quantity of matter and extends to infinity. Therefore, the determinate filling, in accordance with its measure, of a space by matter,^q can in the end be effected only by the attraction of matter extending to infinity, and imparted to each matter in accordance with the measure of its repulsive force.

The *action* of the universal attraction immediately exerted by each matter on all matters, and at all distances, is called *gravitation*; the tendency to move in the direction of greater gravitation is *weight*. The action of the general^r repulsive force of the parts of every given matter is called its *original elasticity*. Hence this property and weight constitute the sole universal characteristics of matter, which are comprehensible a priori, the former

^p *aller Weltmaterie.*

^q *die dem Maße nach bestimmte Erfüllung eines Raumes durch Materie.*

^r *durchgängig.*

internally, and the latter in external relations. For the possibility of matter itself rests on these two properties. *Cohesion*, if this is explicated as the mutual attraction of matter limited solely to the condition of contact, does not belong to the possibility of matter in general, and cannot therefore be cognized a priori as bound up with this. This property would therefore not be metaphysical but rather physical, and so would not belong to our present considerations.

Remark 1

Yet I cannot forbear adding a small preliminary suggestion on behalf of the attempt at such a perhaps possible construction.

[519] (1) Of any force that acts immediately at various distances, and is limited, as to the degree with which it exerts moving force on any given point at a certain distance, only by the magnitude of the space into which it must diffuse so as to act on this point, one can say that in all the spaces, large or small, into which it diffuses, it always constitutes an equal quantum, but [also] that the degree of its action on that point in this space is always in inverse ratio to the space, into which it has had to diffuse, so that it could act on this point. Thus light, for example, diffuses from an illuminating point in all directions on spherical surfaces, which constantly increase with the squares of the distance, and the quantum of illumination on all of these spherical surfaces, which become greater to infinity, is always the same in total. But it follows from this that a given equal part of one of these spherical surfaces must become ever less illuminated with respect to its degree, as the surface of diffusion of precisely the same light quantum becomes greater. And so, too, with all other forces, and the laws whereby they must diffuse, either on surfaces or in volumes,^s so as to act on distant objects in accordance with their nature. It is better to represent the diffusion of a moving force at all distances from a point in this way, rather than as is customary, for example, in optics, by means of rays diverging from one another radially from a central point. For since, as an unavoidable consequence of their divergence, lines drawn in this way can never fill the space through which they spread, nor the surfaces on which they fall, no matter how many are drawn or plotted, they give rise only to troublesome inferences, and these in turn to hypotheses, which might well be avoided

^s *körperlichen Raum*.

by merely taking into consideration the magnitude of the whole spherical surface – which is to be *uniformly* illuminated by the same quantity of light; and the degree of its illumination at every place is then naturally taken in inverse ratio of its magnitude to the whole, and similarly for any other diffusion of a force through spaces of different magnitudes.

(2) If the force is an immediate attraction at a distance, then it is even more necessary to represent the directed lines of attraction, not as if they diverged like rays from the attracting point, but rather as converging from every point of the surrounding spherical surface (whose radius is the given distance) towards it. For the very directed line of motion towards the point, which is the cause and goal of this motion, already yields the terminus a quo from which the lines must begin, namely, from every point of the surface from which they take their direction towards the attracting central point, and not conversely. For this magnitude of the surface alone determines the aggregate of lines; the central point leaves this undetermined.*

(3) If the force is an immediate repulsion, by which a point (in the merely [520] mathematical presentation) fills a space *dynamically*, and the question is by what law of infinitely small distances (which here count as equivalent to contacts) an original repulsive force (whose limitation thus rests simply on the space in which it is diffused) acts at various distances, then

* It is impossible by lines radiating from a point to represent surfaces at given distances as completely [519] filled with their action, whether of illumination or attraction. Thus, in the case of such diverging [520] light rays, the lesser illumination of a distant surface would rest merely on the circumstance that between the illuminated places remain unilluminated ones, and the more distant the surface the larger they are. Euler's hypothesis avoids this impropriety, but has all the more difficulty in making the rectilinear motion of light conceivable. Yet this difficulty flows from an easily avoidable mathematical representation of light matter as an agglomeration of little spheres, which would certainly yield a lateral motion of light in accordance with their varying obliquity to the direction of impact. Instead of this, however, there is no obstacle to thinking the matter in question as an original fluid, and, indeed, as fluid throughout, without being divided into rigid particles. If the mathematician wants to make intuitive the decrease of light at increasing distances, he uses rays diverging radially to represent the magnitude of the space on the spherical surface of its diffusion, wherein the same quantity of light is supposed to be uniformly diffused between these rays, and thus to represent the decrease of the degree of illumination. But he does not want one to view these rays as the only sources of illumination, as if places empty of light, which would be greater at greater distances, were always to be found between them. If one wants to imagine each such surface as illuminated throughout, then the same quantity of illumination as covers the smaller surfaces must be thought as uniformly [spread] over the larger surfaces; so, in order to indicate the rectilinear direction, straight lines from the surface, and all of its points, must be drawn toward the illuminating point. The action and its quantity must be thought of beforehand, and the cause thereupon specified. Precisely the same holds for rays of attraction, if one wants to call them that, and indeed, for all directions of forces that are supposed to fill a space, and even a volume, proceeding from a point.

it is even less possible to make this force representable by diverging rays of repulsion from the assumed repelling point, even though the direction of motion has this point as its terminus a quo. For the space into which the force must be diffused in order to act at a distance is a volume, which is supposed to be thought as filled. (The manner in which a point could do this by moving force, that is, dynamically fill a volume, is certainly not capable of further mathematical presentation.) And diverging rays from a point cannot possibly make representable the repulsive force of a filled volume. Rather, one would simply estimate the repulsion, at various infinitely small distances of these mutually impelling points, as [521] merely in inverse ratio to the volumes that each of them fills dynamically, and thus to the cube of their distances, without being able to construct it.

(4) Thus the original attraction of matter would act in inverse ratio to the squares of the distance at all distances, the original repulsion in inverse ratio to the cubes of the infinitely small distances, and, through such an action and reaction of the two fundamental forces, matter filling its space to a determinate degree would be possible. For since repulsion increases with the approach of the parts to a greater extent than attraction, the limit of approach, beyond which no greater is possible by the given attraction, is thereby determined, and so is that degree of compression which constitutes the measure of the intensive filling of space.

Remark 2

I am well aware of the difficulty in this mode of explaining the possibility of a matter in general. It consists in this, that if a point cannot immediately propel another by repulsive force, without at the same time filling the entire volume up to the given distance with its force, then it appears to follow that this volume would have to contain several impelling points, which contradicts the presupposition, and was refuted above (Proposition 4) under the name of a sphere of repulsion of the simple [elements] in space. But there is a difference between the concept of an actual space, which can be given, and the mere idea of a space, which is thought simply for determining the ratio of given spaces, but is not in fact a space. In the case put forward, of a supposed physical monadology, there were supposed to be actual spaces filled dynamically by a point, namely, through

repulsion;³² for they would exist as points prior to any possible generation of matter therefrom, and would determine, through their own spheres of activity, that part of the space to be filled which could belong to them. So on this hypothesis, matter cannot be viewed as divisible to infinity, and as quantum continuum. For the parts that immediately repel one another have a determinate distance from one another (the sum of the radii of the spheres of their repulsion). By contrast, if, as is actually the case, we think matter as a continuous quantity, there is then no distance at all between the points immediately repelling one another, and thus no increasing or decreasing sphere of their activity. But matters can expand or be compressed (like air), and here one does represent to oneself a distance of their adjacent parts, which can increase and decrease. Yet since the adjacent parts of a *continuous* matter are in contact with one another, whether it is further expanded or compressed, one then thinks [522] these distances as *infinitely small*, and this infinitely small space as filled by its repulsive force to a greater or lesser degree. But the infinitely small intervening space is not at all different from contact. Hence it is only the idea of a space, which serves to make intuitive the enlargement of a matter as a continuous quantity, although it cannot, in fact, be actually conceived in this way. If it is said, therefore, that the repulsive forces of the parts of matter that immediately impel one another stand in inverse ratio to the cubes of their distances, this means only that they stand in inverse ratio to the volumes one imagines between parts that are nevertheless in immediate contact, and whose distance must for precisely this reason be called *infinitely small*, so as to be distinguished from every actual distance. Hence one must not object to a concept itself because of difficulties in constructing it, or, rather, because of a misunderstanding of this construction. For otherwise it would apply to the mathematical presentation of the proportion in accordance with which attraction takes place at various distances, no less than that whereby every point in an expanding or contracting whole of matter immediately repels the others. The universal law of dynamics would in both cases be this: the action of the moving force, exerted by a point on every other point external to it, stands in inverse ratio to the space into which the same quantum of

³² In the *Physical Monadology* of 1756 Kant formulates the law of repulsion as in inverse ratio to the cube of the (*finite*) distance rather than as in inverse ratio to the cube of the *infinitely small* distance. See the Scholium to Proposition X (Ak 1:484–85). (So “actual space” means *finite space* in the *Metaphysical Foundations*.) Compare also note 23 above.

moving force would need to have diffused, in order to act immediately on this point at the determinate distance.

From the law of the parts of matter repelling one another originally in inverse cubic ratio to their infinitely small distances, a law of expansion and contraction of matter completely different from Mariotte's law for the air must therefore necessarily follow; for the latter proves fleeing forces of its adjacent parts standing in inverse ratio to their distances, as Newton demonstrates (*Principia*, Book II, Prop. 23, Schol.).³³ But we may also view the expansive force of air, not as the action of *originally* repelling forces, but as resting rather on *heat*, which compels the proper parts of air (to which, moreover, actual distances from one another are attributable) to flee one another, not merely as a matter penetrating it, but rather, to all appearances, through its vibrations. But that these tremors must impart a fleeing force to the adjacent parts, standing in inverse ratio to their distances, can doubtless be made conceivable in accordance with the laws of communication of motion through the oscillation of elastic matters.

[523] I declare, furthermore, that I do not want the present exposition of the law of an original repulsion to be viewed as necessarily belonging to the goals of my metaphysical treatment of matter. Nor do I want this latter (for which it is enough to have presented the filling of space as a dynamical property of matter) to be mixed up with the conflicts and doubts that could afflict the former.

General Note to Dynamics

If we look back over all our discussions of the subject, we will notice that we have therein considered the following: *first*, the *real* in space (otherwise called the solid), in the filling of space through *repulsive force*; *second*, that which in relation to the first, as the proper object of our outer perception, is *negative*, namely, *attractive force*, whereby, for its own part, all space would be penetrated, and thus the solid would be completely destroyed; *third*, the *limitation* of the first force by the second, and the determination of the *degree of filling* of a space that rests on this. Hence, the *quality* of

³³ At issue is what is now known as Boyle's law: $PV = \text{constant}$. Newton proves in Proposition 23 of Book II that in a fluid composed of particles repelling one another, the force will be inversely as the distance if the density (that is, mass over volume) is as the compression, and conversely. See Cohen and Whitman, pp. 697–99.

matter, under the headings of *reality*, *negation*, and *limitation*, has been treated completely, so far as pertains to a metaphysical dynamics.

General Remark to Dynamics

The general principle of the dynamics of material nature is that everything real in the objects of the outer senses, which is not merely a determination of space (place, extension, and figure), must be viewed as moving force. So by this principle the so-called solid or absolute impenetrability is banished from natural science, as an empty concept, and repulsive force is posited in its stead. But the true and immediate attraction, by contrast, is thereby defended against all sophistries of a metaphysics that misunderstands itself, and, as a fundamental force, is declared necessary for the very possibility of the concept of matter. Now from this it follows that space, if it should be necessary, can be assumed to be completely *filled*, and in different degrees, even *without dispersing empty interstices* within matter.³⁴ For, in accordance with the originally different degree of the repulsive forces, on which rests the first property of matter, namely, that of filling a space, their relation to the original attraction (whether of any [piece of] matter separately, or to the united attraction of all matter in the universe) [524] can be thought of as infinitely various. This is because attraction rests on the aggregate of matter in a given space, whereas its expansive force, by contrast, rests on the degree of filling of this space, which can be very different specifically (as the same quantity of air, say, in the same volume, manifests more or less elasticity in accordance with its greater or lesser heating). The general ground for this is that through true attraction *all parts* of a matter act immediately *on every part* of another, whereas through expansive force only those *at the surface of contact* act, so that it is all the same whether much or little of this matter is found behind that surface. Now a great advantage for natural science already arises here, since it is thereby relieved of the burden of fabricating a world from the full and the empty in accordance with mere fantasy. On the contrary, all spaces can be thought of as full, and yet as filled in different measures, whereby empty space at least loses its *necessity*, and is demoted to the value of an hypothesis. For it could otherwise usurp the title of a principle, under

³⁴ See the discussion of this point in the Anticipations of Perception: A173–75/B215–16. (The same point is discussed further below [533–35].)

the pretense of being a necessary condition for explaining the different degrees of the filling of space.

In all this the advantage of a metaphysics that is here used methodically, to get rid of principles that are equally metaphysical, but have not been brought to the test of criticism, is apparently only *negative*. Nevertheless, the field of the natural scientist is thereby indirectly enlarged. For the conditions by which he formerly limited himself, and through which all original moving forces were philosophized away, now lose their validity. But one should guard against going beyond that which makes possible the general concept of a matter as such, and wishing to explain a priori its particular, or even specific, determination and variety. The concept of matter is reduced to nothing but moving forces, and one could not expect anything else, since no activity or change can be thought in space except mere motion. But who pretends to comprehend the possibility of the fundamental forces? They can only be assumed if they unavoidably belong to a concept that is demonstrably fundamental and not further derivable from any other (like that of the filling of space), and these, in general, are repulsive forces and the attractive forces that counteract them. We can indeed certainly judge a priori about the connection and consequences of these forces, whatever relations among them one can think without contradiction, but cannot yet presume to suppose one of them as actual. For to be authorized in erecting an hypothesis, it is unavoidably required that the *possibility* of what we suppose be completely *certain*, but with fundamental forces their possibility can never be comprehended. And here the mathematical-mechanical mode of explanation has an advantage [525] over the metaphysical-dynamical [mode], which cannot be wrested from it, namely, that of generating from a thoroughly homogeneous material a great specific variety of matters, which vary both in density and (if foreign forces are added) mode of action, through the varying shape of the parts and the empty interstices interspersed among them. For the possibility of both the shapes and the empty interstices can be verified with mathematical evidence. By contrast, if the material itself is transformed into fundamental forces (whose laws we cannot determine a priori, and are even less capable of enumerating reliably a manifold of such forces sufficient for explaining the specific variety of matter), we lack all means for *constructing* this concept of matter, and presenting what we thought universally as possible in intuition. Conversely, however, a merely mathematical physics pays doubly for this advantage on the other side. First,

it must take an empty concept (of absolute impenetrability) as basis; and second, it must give up all forces *inherent* in matter; and beyond this, further, with its original configurations of the fundamental material and its interspersing of empty spaces, as the need for explanation requires them, such a physics must allow more freedom, and indeed rightful claims, to the imagination in the field of philosophy than is truly consistent with the caution of the latter.

Instead of a sufficient explanation for the possibility of matter and its specific variety from these fundamental forces, which I cannot provide, I will present completely, so I hope, the moments to which its specific variety must collectively be reducible (albeit not conceivable in regard to its possibility). The remarks inserted between the definitions^t will explain their application.

1. A **body**, in the physical sense, is a *matter between determinate boundaries* (which therefore has a figure). *The space between these boundaries, considered in accordance with its magnitude, is the volume* [of the body].^u The degree of the filling of a space with determinate content^v is called **density**. (Otherwise the term *dense* is also used absolutely for what is *not hollow*, that is, vesicular or porous.) In this sense, there is an absolute density in the system of absolute impenetrability, that is, when a matter contains no empty interstices at all. In accordance with this concept of the filling of space we make comparisons, and call one matter denser than another when it contains less emptiness, until finally that in which no part of the space is empty is called perfectly dense. One can only make use of the latter expression in connection with the merely mathematical concept of matter, but in the dynamical system of a merely relative impenetrability there is no maximum or minimum of density, and yet every matter, however rarefied, can still be called completely dense, if it fills its space entirely without containing empty interstices, and is thus a continuum, [526] not an interruptum. In comparison with another matter, however, it is less dense, in the dynamical sense, if it fills its space entirely, but not to the same degree. But in this system, too, it is inappropriate to think of matters as related with respect to their density, if we do not imagine them as specifically of the same kind, so that one can be generated from the

^t Definitionen. ^u *Raumesinhalt* (volumen).

^v *Der Grad der Erfüllung eines Raumes von bestimmtem Inhalt.*

other by mere compression. Now since the latter [condition] by no means appears to be necessary to the nature of all matter in itself, no comparison with regard to their density can properly take place between matters of different kinds, between water and mercury, for example, even though it is customary.

2. *Attraction, insofar as it is thought merely as active in contact*, is called **cohesion**. (To be sure, it is confirmed by very good experiments that the same force, which in contact is called cohesion, is also found to be active at a very small distance. But attraction is still called cohesion, only *insofar* as I think it merely in contact, in accordance with common experience, where it is hardly ever observed at small distances. Cohesion is commonly taken for an entirely general property of matter, not because one is already led to it by the concept of a matter, but because experience shows it everywhere. But this generality must not be understood *collectively*, as if every matter acted, through this kind of attraction, on every other matter in the universe *at once*, like gravitation; it must rather be understood merely *disjunctively*, as acting, that is, on one matter or another with which it comes into contact, of whatever kind it may be. For this reason, and since such attraction, as various grounds of proof can show, is not a penetrating, but only a surface force; since it is not even determined everywhere in accordance with density; since for full strength of cohesion a prior state of fluidity of the matters and their subsequent rigidification is required, whereby the closest possible contact of broken solid matters at precisely the same surfaces where they previously cohered so strongly, in a cracked mirror, for example, is still very far from permitting any longer that degree of attraction it had gained on rigidification from a fluid state; I therefore take this attraction in contact to be no fundamental force of matter, but only a derivative one; of which more below.) *A matter whose parts, however strong their mutual cohesion, can nonetheless be mutually displaced by every moving force, however small, is fluid. But parts of a matter are so displaced, when they are merely compelled, without reducing the quantum of contact, to interchange such contact. Parts, and thus also matters, are separated, when the contact is not merely exchanged with others, but destroyed or reduced in quantity. A solid – or better a rigid – body (corpus rigidum) is one whose parts cannot be so displaced by every force – and therefore resists displacement with a certain degree of force. – The resistance to such mutual displacement of matters is friction.* The resistance to separation of matters in contact

[527]

is cohesion. Fluid matters therefore undergo no friction when divided; where it occurs, the matters, at least in their smallest parts, are taken to be rigid, in greater or lesser degree, where the latter is called *viscosity*.^w *A rigid body is brittle, when its parts cannot be mutually displaced without breaking apart*, and thus when their cohesion cannot be changed without at the same time being destroyed. (It is quite wrong to locate the difference between fluid and solid matters in the different degree of cohesion of their parts. For to call a matter fluid does not depend on the degree of resistance it opposes to the breaking up of its parts, but only on its opposition to their mutual displacement. The former can be as large as one wishes, but in a fluid matter the latter is still always = 0. Consider a drop of water. If a particle within it is drawn to one side by an attraction, however great, of the neighboring parts that are in contact with it, it is still drawn just as much to the opposite side as well; and since the attractions mutually cancel their effects, the particle is just as easily movable as if it were in empty space. That is, the force that is to move it has no cohesion to overcome, but only the so-called inertia, which it would have to overcome in all matter, even if the latter did not cohere with anything. Thus a small microscopic organism will move just as easily within the drop as if there were no cohesion at all to separate. For it actually has no cohesion of the water to destroy, nor any internal contact thereof to diminish – it needs only to change this contact. But if you imagine that this same small organism wants to work its way out through the external surface of the drop, then it should first be noticed that the mutual attraction of the parts of this water droplet causes them to keep moving until they have attained the greatest contact with one another, and thus the smallest contact with empty space, that is, until they have formed a spherical shape. Now if the insect in question is striving to work its way out beyond the surface of the drop, it must change the spherical shape, and thus create more contact between the water and empty space, and hence less contact of its parts with one another, that is, [it must] diminish the cohesion of these parts. And here the water resists it primarily through its cohesion, but not within [528] the drop, where the mutual contact of the parts is not diminished at all, but only changed into contact with other parts, so that they are not in the least separated, but only displaced. One can also apply to the microscopic organism, and indeed on similar grounds, what Newton says of the light

^w *Klebrigkeit* (*viscositas*).

ray: that it is repulsed, not by dense matter, but only by empty space.³⁵ It is clear, therefore, that the increase in cohesion of the parts of a matter does not impair its fluidity in the least. Water coheres far more strongly in its parts than is commonly believed, when one relies on the experiment of a metal plate pulled off the surface of the water; this settles nothing, because here the water is not torn loose over the whole surface of first contact, but on a much smaller one, which, in fact, it has finally arrived at through the displacement of its parts, as a stick of soft wax, say, can be first drawn out thinner by a hanging weight, and must then rupture at a much smaller surface than was originally assumed. But what is entirely decisive in regard to our concept of fluidity is this: that *fluid* matters can also be defined^x as those, *in which every point endeavors to move in all directions with precisely the same force with which it is pressed towards any one of them*, a property on which rests the first law of hydrodynamics,³⁶ although it can never be attributed to an agglomeration of smooth and yet solid corpuscles, as can be shown by a very easy calculation of its pressure in accordance with the laws of composite motion, thereby proving the original character of the property of fluidity. If the fluid matter were to suffer the least resistance to displacement, and thus even the smallest amount of friction, then the latter would increase with the strength of the pressure by which its parts are pressed against one another, and a pressure would finally obtain at which its parts could not be displaced along one another by any small force. Consider, for example, a bent tube with two arms, one of which may be arbitrarily wide, and the other arbitrarily narrow, so long as it is not a capillary tube;³⁷ if one imagines both arms several hundred feet high, then, according to the laws of hydrostatics, the fluid matter in the narrow arm would stand precisely as high as in the wide one. But since the pressure on the bases of the tubes, and hence also on the part that joins them in common, can be thought as increasing to infinity in proportion to the heights, it follows that if the least amount of friction occurred between the parts of the fluid, a height for the tubes could be

^x *erklärt*.

³⁵ The reference appears to be to Query 29 of the *Opticks*, where Newton speaks of rays of light penetrating glass and falling “obliquely on the *Vacuum*” so as to be “bent backwards into the Glass and totally reflected” (4th edn, 1730: p. 371).

³⁶ This appears to be a misprint for “hydrostatics”: see the text below.

³⁷ That is, a tube so narrow that adhesion to the sides, combined with cohesion and surface tension, can result in a liquid rising in the tube (a kind of “apparent attraction”).

found, at which a small quantity of water, poured into the narrower tube, did not disturb that in the wider one from its place. So the water column in the former would come to stand higher than that in the latter, because the lower parts, at such great pressure against one another, could no longer be displaced by so small a moving force as that of the added weight of water. But this is contrary to experience, and even to the concept of a fluid. The same holds if, instead of pressure by weight, one posits cohesion of the parts, however great one cares to make it. The cited second definition of fluidity, on which rests the fundamental law of hydrostatics – namely, that it is that property of a matter whereby any part of it strives to move in all directions with precisely the same force by which it is pushed in any given direction – follows from the first definition, if one combines it with the principle of general dynamics that all matter is originally elastic. For this matter must then be striving to expand in all directions of the space in which it is compressed, with the same force by which the pressure occurs in any direction, whatever it may be, that is, if the parts of a matter can be displaced along one another by any force, without resistance, as is actually the case with fluids, it must be striving to move in all directions. Hence friction, properly speaking, is attributable only to rigid matters (whose possibility requires yet another ground of explanation besides the cohesion of the parts), and friction already presupposes the property of rigidity. But why certain matters, even though they may have no greater, and perhaps even a lesser force of cohesion than other matters that are fluid, nevertheless resist the displacement of their parts so strongly, and hence can be separated in no other way than by destroying the cohesion of all parts in a given surface at once, which then yields the semblance of a superior cohesion – how, that is, rigid bodies are possible – is still an unsolved problem, no matter how easily the common doctrine of nature presumes to have settled it.) [529]

3. **Elasticity** (spring-force) is the capacity of a matter, *when its magnitude or figure are changed by another moving force, to reassume them again when this latter is diminished*. It is either *expansive* or *attractive* elasticity: one to regain a previously greater volume after compression, the other a previously smaller volume after expansion. (Attractive elasticity is obviously derivative, as the term already shows. An iron wire, stretched by a hanging weight, springs back into its volume when the band is cut. In virtue of the same attraction that is the cause of its cohesion, or, in the case of fluid matters, if heat were suddenly to be extracted from mercury,

the matter would quickly reassume the previously smaller volume. The elasticity that consists only in regeneration of the previous figure is always attractive, as in the case of a bent sword-blade, where the parts, stretched away from one another on the convex surface, strive to reassume their previous proximity, and so a small drop of mercury can likewise be called elastic. Expansive elasticity, however, can be either original or derivative. Thus air has a derivative elasticity in virtue of the matter of heat, which is most intimately united with it, and whose own elasticity is perhaps original. By contrast, the fundamental material of the fluid we call air must nonetheless, as matter in general, already have original elasticity in itself. It is not possible to decide with certainty to which type an observed elasticity belongs in any given case.)

4. *The action of moved bodies on one another by communication of their motion is called **mechanical**; but the action of matters is called **chemical**, insofar as they mutually change, even at rest, the combination of their parts through their inherent forces.* This chemical influence is called **dissolution**, insofar as it *has the separation of the parts of a matter as its effect.* (Mechanical separation, by means of a wedge driven between the parts of a matter, for example, is therefore entirely different from chemical separation, because the wedge does not act by means of inherent force.) But that chemical influence whose effect is to isolate two matters dissolved in one another is **decomposition**. A dissolution of specifically different matters by one another, in which no part of the one is found that would not be united with a specifically different part of the other, in the same proportion as the whole, is *absolute dissolution*, which can also be called *chemical penetration*. Whether the dissolving forces that are actually to be found in nature are capable of effecting a complete dissolution may remain undecided. Here it is only a question of whether such a dissolution can be thought. Now it is obvious that, so long as the parts of a dissolved matter remain small clots (moleculae), a dissolution of them is no less possible than that of the larger parts. Indeed, if dissolving force remains, such a dissolution must actually proceed until there is no longer any part that is not made up of the solvent and the solute, in the same proportion in which the two are found in the whole. Thus, because in such a case there can be no part of the volume of the solution that would not contain a part of the solvent, the latter must fill this volume completely as a continuum. In precisely the same way, because there can be no part of this same volume of the solution that would not contain a proportional part of the solute, the latter

must also fill the whole space constituting the volume of the mixture, as a continuum. But if two matters fill one and the same space, and each of them does this completely, they *penetrate* one another. Hence a complete chemical dissolution would be a penetration of matters, which would nonetheless be entirely different from mechanical penetration. For in the latter case it is thought that, as the moved matters approach one another more closely, the repulsive force of the one can completely surpass that of the other, so that one or both can have their extension shrink to nothing. [531] Here, by contrast, the extension remains, and it is only that the matters together occupy a space, which accords with the sum of their densities, not outside, but inside one another, that is, through intussusception³⁸ (as it is customarily called). It is not easy to make any objection to the possibility of this complete dissolution, and thus chemical penetration, even though it contains a *completed* division to infinity, which, in this case, still involves no contradiction, because the dissolution takes place continuously throughout a time, and thus equally through an infinite series of moments with acceleration. By the division, moreover, the sum of the surfaces of the matters yet to be divided increases, and, since the dissolving force acts continuously, the entire dissolution can be completed in a *specifiable* time. The inconceivability of such a chemical penetration of two matters is to be attributed to the inconceivability of dividing any such continuum in general to infinity. If one recoils from this complete dissolution, then one must assume that it proceeds only up to certain small clots of the solute, which swim in the solvent at given distances from one another, without being able to offer the slightest reason why these clots are not equally dissolved, since they are still always divisible matter. It may always be true in nature, so far as experience reaches, that the solvent acts no further; but all that is at issue here is the possibility of a dissolving force that also dissolves this clot, as well as anything left over from that, until solution is completed. The volume occupied by the solution may be equal to, smaller than, or even greater than the sum of the spaces occupied by the mutually dissolving matters before mixing, depending on the ratio of the attracting forces to the repulsions. In the solution, each matter by itself, and both united, constitute an *elastic medium*, and this, *on its own*,

³⁸ From the Latin *intus* (inside) and *suscipio* (to take up). The term is commonly used for organic processes of nourishment and growth; it is so used by Kant at A833/B861, where he compares the system of pure reason to an animal body that can grow “from inside (per intus susceptionem)” but not “from outside (per appositionem).”

can supply a sufficient reason why the solute does not again separate from the solvent through its weight. For the attraction of the latter, since it takes place equally strongly in all directions, itself destroys the resistance of the solute; and to assume a certain viscosity in the fluid by no means accords with the great force that such dissolved matters, acids diluted with water, for example, exert on metallic bodies. They do not merely lie on them, as would have to happen if they were merely afloat in their medium; rather, they split them up with great attractive force, and spread throughout the entire space of the vehicle. Moreover, even if we supposed that art had no such dissolving forces at its disposal as to effect a complete dissolution, nature, in its vegetable or animal operations, could still perhaps manifest them, and thereby generate matters that, although certainly mixed, can be separated again by no art.³⁹ This chemical penetration could also be found even where one of two matters is not in fact separated by and literally dissolved in the other, as caloric, for example, penetrates bodies; for if it merely dispersed into their empty interstices, the solid substance would itself remain cold, since it could not absorb anything from it. We might even imagine, in this way, an apparently free passage of certain matters through others, for example, of magnetic matter, without preparing, for this purpose, open passages and empty interstices in all matters, even the most dense. Yet here is not the place to uncover hypotheses for particular phenomena, but only the principle in accordance with which they are all to be judged. Everything that relieves us of the need to resort to empty spaces is a real gain for natural science, for they give the imagination far too much freedom to make up by fabrication^y for the lack of any inner knowledge of nature. In the doctrine of nature, the absolutely empty and the absolutely dense are approximately what blind accident and blind fate are in metaphysical science, namely, an obstacle to the governance of reason, whereby it is either supplanted by fabrication or lulled to rest on the pillow of occult qualities.

But now as to the procedure of natural science with respect to the most important of all its tasks – namely, that of explaining a potentially infinite *specific variety of matters* – one can take only two paths in this

^y *Erdichtung*.

³⁹ *Gesetzt auch, daß die Kunst keine chemische Auflösungskräfte dieser Art, die eine vollständige Auflösung bewirkten, in ihrer Gewalt hätte, so könnte doch vielleicht die Natur sie in ihren vegetabilischen und animalischen Operationen beweisen und dadurch vielleicht Materien erzeugen, die, ob sie zwar gemischt sind, doch keine Kunst wiederum scheiden kann.* A traditional term for chemistry is *Scheidekunst*.

connection: the *mechanical*, by combination of the absolutely full with the absolutely empty, and an opposing *dynamical* path, by mere variety in combining the original forces of repulsion and attraction to explain all differences of matters. The first has as materials for its derivation *atoms* and the *void*. An atom is a small part of matter that is physically indivisible. A matter is physically *indivisible* when its parts cohere with a force that cannot be overpowered by any moving force in nature. An atom, insofar as it is specifically distinguished from others by its figure, is called a *primary particle*. A body (or particle) whose moving force depends on its figure is called a *machine*. The mode of explaining the specific variety of matters by the constitution and composition of their smallest parts, as machines, is the *mechanical natural philosophy*. But that which derives this specific variety from matters, not as machines, that is, mere instruments of external moving forces, but from the moving forces of attraction and repulsion originally inherent in them, can be called the *dynamical natural philosophy*. The mechanical mode of explanation, since it is the most tractable for mathematics, has, under the name of *atomism* [533] or the *corpuscular philosophy*, always retained its authority and influence on the principles of natural science, with few changes from Democritus of old, up to Descartes, and even to our time. What is essential therein is the presupposition of the *absolute impenetrability* of the primitive matter, the *absolute homogeneity* of this material, leaving only differences in the shape, and the *absolute insurmountability* of the cohesion of matter in these fundamental particles themselves. These were the *materials* for generating specifically different matters, so as not only to have at hand an invariable, and, at the same time, variously shaped fundamental material for explaining the invariability of species and kinds, but also to explain *mechanically*, from the shapes of these primary parts, as machines (where nothing further is lacking but an external impressed force), the manifold workings of nature. But the first and foremost authentication for this system rests on the apparently unavoidable *necessity* for using *empty spaces on behalf of the specific difference in the density* of matters. These spaces were taken to be distributed within the matters, and between these particles, in any proportion found necessary, so great, indeed, for the sake of some phenomena, that the filled part of the volume of even the densest matter is virtually negligible relative to the empty part. – In order now to introduce a dynamical mode of explanation (which is much more appropriate and conducive to experimental philosophy, in that it leads us directly to the

discovery of matter's inherent moving forces and their laws, while restricting our freedom to assume empty interstices and fundamental particles of determinate shapes, neither of which are determinable or discoverable by any experiment), it is not at all necessary to frame new hypotheses. It is only necessary to refute the postulate of the merely mechanical mode of explanation – namely, *that it is impossible to think a specific difference in the density of matters without interposition of empty spaces* – by simply advancing a mode of explanation in which this can be thought without contradiction. For once the postulate in question, on which the merely mechanical mode of explanation rests, is shown to be invalid as a principle, then it obviously does not have to be adopted as an hypothesis in natural science, so long as a possibility remains for thinking the specific difference in densities even without any empty interstices. But this necessity⁴⁰ rests on the circumstance that matter does not fill its space (as merely mechanical natural scientists assume) by absolute impenetrability, but rather by repulsive force, which has a degree that can be different in different matters; and, since in itself it has nothing in common with the attractive force, which depends on the quantity of matter, it may be *originally different* in degree in different matters whose attractive force is the same. Thus the degree of expansion of these matters, when the quantity is the same, and, conversely, the quantity of matter at the same volume, that is, its density, originally admit of very large specific differences. In this way, one would not find it impossible to think a matter (as one imagines the aether, for example) that completely filled its space without any emptiness, and yet with an incomparably smaller quantity of matter, at the same volume, than any bodies we can subject to our experiments. In the aether, the repulsive force must be thought as incomparably larger in proportion to its inherent attractive force than in any other matters known to us. And this, then, is the one and only assumption that we make, simply *because it can be thought*, but only to controvert an hypothesis (of empty spaces), which rests solely on the pretension that such a thing *cannot be thought* without empty spaces. For, aside from this, no law of either attractive or repulsive force may be risked on a priori conjectures. Rather, everything, even universal attraction as the cause of weight, must be inferred, together with its laws, from data of experience. Still less may such laws be attempted for chemical affinities otherwise than by way of experiments. For it lies

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⁴⁰ It would appear, rather, that “possibility” is intended here.

altogether beyond the horizon of our reason to comprehend original forces a priori with respect to their possibility; all natural philosophy consists, rather, in the reduction of given, apparently different forces to a smaller number of forces and powers that explain the actions of the former, although this reduction proceeds only up to fundamental forces, beyond which our reason cannot go. And so metaphysical investigation behind that which lies at the basis of the empirical concept of matter is useful only for the purpose of guiding natural philosophy, so far as this is ever possible, to explore dynamical grounds of explanation. For these alone permit the hope of determinate laws, and thus a true rational coherence of explanations.

This is now all that metaphysics can ever achieve towards the construction of the concept of matter, and thus to promote the application of mathematics to natural science, with respect to those properties whereby matter fills a space in a determinate measure – namely, to view these properties as dynamical, and not as unconditioned original positings,^z as a merely mathematical treatment might postulate them.

[535] The well-known question as to the admissibility of empty spaces in the world may serve as our conclusion. The *possibility* of such spaces cannot be disputed. For space is required for all forces of matter, and, since it also contains the conditions of the laws of diffusion of these forces, it is necessarily presupposed prior to all matter. Thus attractive force is attributed to matter insofar as it *occupies* a space around itself, through attraction, without at the same time *filling* this space. Thus this space can be thought as empty, even where matter is active, because matter is not active there by repulsive forces, and hence does not fill this space. But no experience, or inference therefrom, or necessary hypothesis for their explanation, can justify us in assuming empty spaces as *actual*. For all experience yields only comparatively empty spaces for our cognition, which can be completely explained, to any arbitrary degree, by the matter's property of filling its space with greater or infinitely diminishing expansive force, without requiring empty spaces.

^z *Positionen*.

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Third Chapter Metaphysical Foundations of Mechanics

Explication 1

Matter is the movable insofar as it, as such a thing, has moving force.

Remark

This is now the third definition of matter. The merely dynamical concept could consider matter also as at rest; for the moving force there dealt with had merely to do with the filling of a certain space, without the matter filling it needing to be seen as itself moved.^a Repulsion was therefore an originally moving force for *imparting* motion. In mechanics, by contrast, the force of a matter set in motion is considered as *communicating* this motion to another. It is clear, however, that the movable would have no moving force *by means of its motion*, if it did not possess originally moving forces, by which it is active in every place where it is found, prior to any inherent motion of its own.^b No matter would impress proportionate motion on another matter lying straight *ahead* and in the way of its motion, if both did not possess original laws of repulsion; nor could a matter, by its motion, compel another *to follow* straight *behind* it (to drag it along behind), if both did not possess attractive forces. Thus all mechanical

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^a ohne daß die Materie, die ihn erfüllte, selbst als bewegt angesehen werden dürfte.

^b vor aller eigener Bewegung.

with which, it acts immediately in its motion, and thereby communicates its own inherent motion to another. I will be forgiven if I do not here further discuss the communication of motion by attraction (for example, if a comet, perhaps, with stronger attractive power than the earth, were to drag the latter in its wake in passing ahead of it), but only that by means of repulsive forces, and thus by pressure (as by means of tensed springs), or through impact. For, in any event, the application of the laws of the one case to those of the other differs only in regard to the line of direction, but is otherwise the same in both cases.

Explication 2

The *quantity of matter* is the aggregate of the movable in a determinate space. Insofar as all its parts are considered as acting (moving) together in their motion, it is called *mass*, and one says that a matter *acts in mass*, when all its parts, moved in the same direction, *together* exert their moving force externally. A mass of determinate shape is called a *body* (in the mechanical meaning). The *quantity of motion* (estimated mechanically) is that which is estimated by the quantity of the moved matter and its speed together; *phoronomically* it consists merely in the degree of speed.

Proposition 1

The quantity of matter, in comparison with *every* other matter, can be estimated only by the quantity of motion at a given speed.

Proof

Matter is infinitely divisible. So its quantity cannot be immediately determined by an aggregate of its parts. For even if this occurs in comparing the given matter with another of the same kind, in which case the quantity of matter is proportional to the size of the volume, it is still contrary to the requirement of the proposition, that it is to be estimated in comparison with every other (including the specifically different). Hence matter cannot be validly estimated, either immediately or mediately, in comparison with every *other*, so long as we abstract from its own inherent motion; no other generally valid measure remains, therefore, except the quantity of its motion. But here the difference of motion, resting on the differing

quantity of matters, can be given only when the speed of the compared matters is assumed to be the same; hence, etc.

Note

The quantity of motion of bodies is in compound ratio to that of the quantity of their matter and their speed, that is, it is one and the same whether I make the quantity of matter in a body twice as large, and retain the same speed, or double the speed, and retain precisely this mass. For the determinate concept of a quantity is possible only through the construction of the quantum. But in regard to the concept of quantity, this is nothing but the *composition* of the equivalent; so construction of the quantity of a motion is the composition of many motions equivalent to one another. Now according to the phoronomical propositions, it is one and the same whether I impart to a single movable a certain degree of speed, or to each of many movables all smaller degrees of speed, resulting from the given speed divided by the aggregate of movables. From this first arises a seemingly phoronomical concept of the quantity of a motion, as composed of many motions of movable points, external to one another yet united in a whole. If these points are now thought as something that has moving force *through its motion*, then there arises from this the mechanical concept of the quantity of motion. In phoronomy, however, it is not appropriate to represent a motion as composed of many motions *external to one another*, since the movable, as it is here represented as devoid of

[539] moving force, yields no other difference in the quantity of motion, in any composition with several of its kind, than that which consists merely in speed. As the quantity of motion in a body relates to that of another, so also does the magnitude of their action, but this is to be understood as the *entire* action. Those who merely took the quantity of a space filled with resistance as the measure of the entire action (for example, the height to which a body with a certain speed can rise against gravity, or the depth to which it can penetrate into soft matters) came out with another law of moving forces for *actual* motions – namely, that of the compound ratio of the quantity of matters and the squares of their speeds. But they overlooked the magnitude of action in the given time, during which the body traverses its space at a lower speed; and yet this alone can be the measure of a motion that is exhausted by a given uniform resistance. Hence there can be no difference, either, between living and dead forces, if the moving

forces are considered mechanically, that is, as those which bodies have insofar as they themselves are moved, whether the speed of their motion be finite or infinitely small (mere striving towards motion). Rather, it would be much more appropriate to call dead forces those, such as the original moving forces of dynamics, whereby matter acts on another, even when we abstract completely from its own inherent motion, and also even from its striving to move; by contrast, one could call living forces all mechanical moving forces, that is, those moving by inherent motion, without attending to the difference of speed, whose degree may even be infinitely small – if in fact these terms for dead and living forces still deserve to be retained.⁴¹

Remark

In order to avoid prolixity we will merge the explanation of the previous three statements into one remark.

That the quantity of matter can only be thought as the aggregate of movables (external to one another), as the definition expresses it, is a remarkable and fundamental proposition of general mechanics. For it is thereby indicated that matter has no other magnitude than that consisting in the *aggregate of manifold [elements] external to one another*, and hence has no *degree of moving force* at a given speed that would be independent of this aggregate, and could be considered merely as intensive magnitude – [540] which would be the case, however, if matter consisted of monads, whose reality in every relation must have a degree that can be larger or smaller, without depending on an aggregate of parts external to one another. As to the concept of mass in this same explication, one cannot take it in the customary way to be the same as that of quantity [of matter]. Fluid matters can act by their own inherent motion in a mass,^c but they can also act as a fluid.^d In the so-called water hammer the impulsive water acts

^c in Masse. ^d im Flusse.

⁴¹ At issue here is the *vis viva* controversy concerning whether mv or mv^2 is the proper measure of moving force – a controversy Kant attempts to mediate in his first published work, *Thoughts on the True Estimation of Living Forces* (1747). By “actual motion” Kant refers to finite speed v as opposed to infinitesimal or “infinitely small” speed dv , and the terms “dead force” and “living force” are Leibnizean terms for mdv and mv^2 receptively. If one integrates mdv/dt with respect to space, one obtains what is now called mechanical work or kinetic energy, $1/2mv^2$. But if one integrates mdv/dt with respect to time, one obtains momentum, mv . So Kant is here siding unequivocally with mv as the proper measure of (mechanical) moving force.

in a mass, that is, with all its parts together.⁴² The same thing happens when water enclosed in a vessel presses down with its weight on the scale on which it stands. By contrast, the water of a millstream does not act on the paddle of an undershot waterwheel in a mass, that is, with all its parts impinging on this paddle together, but only one after the other.⁴³ Thus, if the quantity of matter, which is moved with a certain speed, and has moving force, is to be determined here, one must first look for the *water body*, that is, that quantity of matter which, if it acts in a mass with a certain speed (with its weight), can bring about the same effect. So we also customarily understand by the word *mass* the quantity of matter in a *solid body* (the vessel in which a fluid is contained can also stand proxy for its solidity). Finally, there is something peculiar in the Proposition together with its appended Note. According to the former the quantity of matter must be estimated by the quantity of motion at a given speed, but according to the latter the quantity of motion (of a body, for that of a point consists merely in the degree of speed) must, at the same speed, in turn be estimated by the quantity of the matter moved. And this seems to revolve in a circle, and to promise no determinate concept from either the one or the other. This alleged circle would be an actual one, if it were a reciprocal derivation of two identical concepts from one another. But it contains only the explication of a concept, on the one hand, and that of its application to experience, on the other. The quantity of the movable in space is the quantity of matter; but this quantity of matter (the aggregate of the movable) *manifests itself*^e in experience only by the quantity of motion at equal speed (for example, by equilibrium^f).

It is to be noted, further, that the quantity of matter is the *quantity of substance* in the movable, and thus not the magnitude of a certain quality of the movable (the repulsion or attraction that are cited in dynamics), and that the quantum of substance here means nothing else but the mere

^e *beweiset sich.* ^f *Gleichgewicht.*

⁴² The water hammer consists of a liquid (usually water) hermetically sealed in a glass tube from which all the air has been removed (commonly, by boiling the liquid so that the resulting steam forces the air out and then sealing the tube). If such a tube is inverted, the liquid, due to the lack of intervening air, then rushes immediately to the other end and strikes it quite forcefully – resulting in a loud noise and sometimes the breaking of the tube.

⁴³ An undershot waterwheel (*unterschlägiger Wasserrad*) is rotated by water flowing underneath, whereas overshot wheels are rotated by the weight of water collected above in buckets. Experiments by John Smeaton in 1759 suggested that overshot wheels operate, on average, at double the efficiency of undershot wheels.

aggregate of the movable that constitutes matter. For only the aggregate of the moved can yield, at the same speed, a difference in the quantity of motion. But that the moving force a matter has *in its own inherent* motion [541] alone manifests^g the quantity of *substance*, rests on the concept of the latter as the ultimate *subject* in space (which is in turn no predicate of another) – which, for precisely this reason, can have no other magnitude than that of the aggregate of homogeneous [elements] external to one another. Now since the *inherent motion* of matter is a predicate that determines its subject (the movable), and indicates in a matter, as an aggregate of movables, a plurality of the subjects moved (at the same speed and in the same way), which is not the case for dynamical properties, whose magnitude can also be that of the action of a single subject (where an air particle, for example, can have more or less elasticity); it therefore becomes clear how the quantity of substance in a matter has to be estimated mechanically only, that is, by the quantity of its own inherent motion, and not dynamically, by that of the original moving forces.⁴⁴ Nevertheless, *original attraction*, as the cause of universal gravitation, can still yield a measure of the quantity of matter, and of its substance (as actually happens in the comparison of matters by weighing), even though a dynamical measure – namely, attractive force – seems here to be the basis, rather than the attracting matter’s own inherent motion.⁴⁵ But since, in the case of this force, the action of a matter with all its parts is exerted immediately on all parts of another, and hence (at equal distances) is obviously proportional to the aggregate of the parts, the attracting body also thereby imparts to itself a speed of its own inherent motion (by the resistance of the attracted body), which, in like external circumstances, is exactly proportional to the aggregate of its parts; so the estimation here is still in fact mechanical, although only indirectly so.

Proposition 2

First Law of Mechanics. In all changes of corporeal nature the total quantity of matter remains the same, neither increased nor diminished.

^g *beweise*.

⁴⁴ See the Remark to Explication 5 of the Dynamics [503].

⁴⁵ Here we are concerned with *gravitational* mass in the sense of notes 28 and 31 above. The procedure of *weighing* (in a balance, say) depends on the equality of inertial mass with *passive* gravitational mass. Measuring the mass of the planets by the gravitational attraction on their satellites, by contrast, depends on the equality of inertial mass with *active* gravitational mass.

Proof

(From general metaphysics we take as basis the proposition that in all changes of nature no substance either arises or perishes, and here it is only shown what substance shall be in matter.)⁴⁶ In every matter the movable in space is the ultimate subject of all accidents inhering in matter, and the aggregate of these movables, external to one another, is the quantity of substance. Hence the quantity of matter, with respect to its substance, is nothing else but the aggregate of substances of which it consists. Therefore, the quantity of matter cannot be increased or diminished except in such a way that new substance thereof arises or perishes. Now substance never arises or perishes in any change of matter; so the quantity of matter is also neither increased nor diminished thereby, but remains always the same, and, indeed, as a whole – in such a way, that is, that somewhere in the world it persists in the same quantity, although this or that matter can be increased or diminished, through addition or separation of parts.

Remark

What is essential in this proof to the characterization of the *substance* that is possible only in space, and in accordance with its condition, and thus possible only as object of the *outer* senses, is that its quantity cannot be increased or diminished without substance arising or perishing. For, since all quantity of an object possible merely in space must consist of *parts external to one another*, these, if they are real (something movable), must therefore necessarily be substances. By contrast, that which is considered as object of inner sense can have a magnitude, as substance, which *does not consist of parts external to one another*; and its parts, therefore, are not substances; and hence their arising or perishing need not be the arising or perishing of a substance; and their augmentation or diminution, then, is possible without violating the principle of the persistence of substance. So *consciousness*, and thus the clarity of representations in my soul, and therefore the faculty of consciousness, apperception, and even, along with this, the very substance of the soul, have a *degree*, which can be greater

⁴⁶ In the second edition of the *Critique* Kant changed the statement of the First Analogy to read: "In all change of the appearances substance persists [*beharrt*], and the quantum of substance in nature is neither increased nor diminished" (B224).

or smaller, without any substance at all needing to arise or perish for this purpose. But since, from its gradual diminution, the complete disappearance of the faculty of apperception would finally have to result, the very substance of the soul would still be subject to a gradual perishing, even if it were of a simple nature; for this disappearance of its fundamental force could result, not by division (separation of substance from a composite), but rather, as it were, by expiration^h – and this, too, not in a moment, but by a gradual waning of its degree, whatever the cause of this might be.⁴⁷ The *I*, the general correlate of apperception, and itself merely a thought, designates, as a mere prefix, a thing of undetermined meaning – namely, the subject of all predicates – without any condition at all that would distinguish this representation of the subject from that of a something in general: a substance, therefore, of which, by this term, one has [543] no concept of what it may be. By contrast, the concept of a matter as substance is the concept of the movable *in space*. It is therefore no wonder if the persistence of substance can be proved of the latter, but not of the former, since, in the case of matter, it already results from its *concept* – namely, that it is the movable, which is possible only in space – that what has quantity therein contains a plurality of the real *external to one another*, and thus a plurality of substances; and hence the quantity of matter can be diminished only by division, which is not disappearance – and the latter would also be impossible in matter according to the law of continuity. The thought *I*, by contrast, is *no concept* at all, but only inner perception, and so nothing at all can be inferred from it (except for the total distinctness of an object of inner sense from that which is thought merely as object of the outer senses) – including, in particular, the persistence of the soul as substance.

Proposition 3

Second Law of Mechanics. Every change in matter has an external cause. (Every body persists in its state of rest or motion, in the same direction, and with the same speed, if it is not compelled by an external cause to leave this state.)

^h *Erlöschen.*

⁴⁷ Compare the “Refutation of Mendelssohn’s Proof of the Permanence of the Soul” in the second edition *Paralogisms* (B413–15).

Proof

(From general metaphysics we take as basis the proposition that every change has a *cause*, and here it is only to be proved of matter that its change must always have an *external cause*.)⁴⁸ Matter, as mere object of the outer senses, has no other determinations except those of external relations in space, and therefore undergoes no change except by motion. With respect to the latter, as change of one motion into another, or of a motion into rest, or conversely, a cause must be found (by the principle of metaphysics). But this cause cannot be internal, for matter has no essentially internal determinations or grounds of determination. Hence every change in a matter is based on external causes (that is, a body persists, etc.).

[544] *Remark*

This mechanical law must alone be called the law of *inertia* (*lex inertiae*); the law of an equal and opposite reaction for every action cannot bear this name. For the latter says what matter does, but the former only what it does not do, which is more appropriate to the term *inertia*. The inertia of matter is, and means, nothing else than its *lifelessness*, as matter in itself. *Life* is the faculty of a *substance* to determine itself to act from an *internal principle*, of a *finite substance* to change, and of a *material substance* [to determine itself] to motion or rest, as change of its state. Now we know no other internal principle in a substance for changing its state except *desiring*, and no other internal activity at all except *thinking*, together with that which depends on it, the *feeling* of pleasure or displeasure, and *desire* or willing. But these actions and grounds of determination in no way belong to representations of the outer senses, and so neither [do they belong] to the determinations of matter as matter. Hence all matter, as such, is *lifeless*. The principle of inertia says this, and nothing more. If we seek the cause of any change of matter in life, we will have to seek it

⁴⁸ Compare §V of the Introduction to the *Critique of Judgement*: “A transcendental principle is that through which is represented a priori the universal cognition under which alone things can be objects of our cognition in general. By contrast, a principle is called metaphysical if it represents a priori the condition under which alone objects, whose concept must be empirically given, can be further determined a priori. Thus, the principle of the cognition of bodies as substances and as changeable substances is transcendental, if it is thereby asserted that their changes must have a cause; it is metaphysical, however, if it is thereby asserted that their changes must have an *external cause*” (Ak 5:181).

forthwith in another substance, different from matter, yet combined with it. For in natural knowledge we first have to be acquainted with the laws of matter, as such a thing, and to purge them from the admixture of all other active causes, before we connect them with these latter, in order properly to distinguish how, and in what manner, each of them acts in itself alone. The possibility of a proper natural science rests entirely and completely on the law of inertia (along with that of the persistence of substance). The opposite of this, and thus also the death of all natural philosophy, would be *hylozoism*. From this very same concept of inertia, as mere *lifelessness*, it follows at once that it does not mean a *positive striving* to conserve its state. Only living beings are called inert in this latter sense, because they have a representation of another state, which they abhor, and against which they exert their power.

Proposition 4

Third mechanical law. In all communication of motion, action and reaction are always equal to one another.

Proof

(From general metaphysics we must borrow the proposition that all external action in the world is *interaction*. Here, in order to stay within the bounds of mechanics, it is only to be shown that this interaction (actio [545] mutua) is at the same time *reaction* (reactio); but here I cannot wholly leave aside this metaphysical law of community, without detracting from the completeness of the insight.)⁴⁹ All *active* relations of matters *in space*, and all changes of these relations, insofar as they may be *causes* of certain actions or effects,ⁱ must always be represented as mutual; that is, because all change of matter is motion, we cannot think any motion of a body in relation to another *absolutely at rest* that is thereby also to be set in

ⁱ *Wirkungen* (actions or effects).

⁴⁹ In the second edition of the *Critique* Kant changed the title of the Third Analogy to the “principle of simultaneity [*Zugleichsein*], in accordance with the law of interaction or community” and changed its statement to read: “All substances, in so far as they can be perceived as simultaneous [*zugleich*] in space, are in thoroughgoing interaction.”

motion. Rather, the latter must be represented as only *relatively at rest* with respect to the space that we relate it to, but as moved, together with this space, in the opposite direction, with precisely the same quantity of motion in absolute space as the moved body there has towards it. For the change of relation (and thus the motion) between the two is completely mutual; as much as the one body approaches every part of the other, by so much does the other approach every part of the first. And, since it is here a question, not of the empirical space surrounding the two bodies, but only of the line lying between them (in that they are considered simply in relation to one another, in accordance with the influence that the motion of the one can have on the change of state of the other, abstracting from all relation to the empirical space), their motion is therefore considered as determinable merely in absolute space, in which each of the two must have an equal share in the motion that is ascribed to one of them in relative space, in that there is no reason to ascribe more of this motion to one than the other. On this basis,^j the motion of a body A with respect to another body B at rest, in regard to which it can thereby be moving, is reduced to absolute space; that is, as a relation of acting causes merely related to one another, this motion is so considered that both have an equal share in the motion which, in the appearance, is ascribed to body A alone. And the only way this can happen is that the speed ascribed in relative space to body A alone is apportioned between A and B in inverse ratio to their masses – to A alone its speed in absolute space, and to B, *together with the relative space* in which it is at rest, its speed in the opposite direction. The same appearance of motion is thereby perfectly maintained, but the action in the community of the two bodies is constructed as follows.

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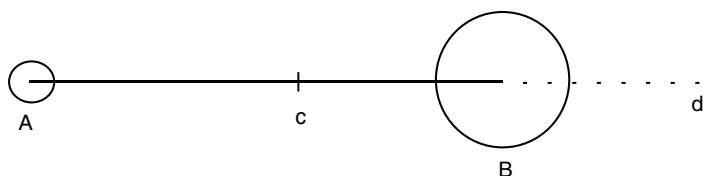


Figure 5

^j Auf diesem Fuß.

Let a body A be approaching the body B, with a speed = AB with respect to the relative space in which B is *at rest*. One divides the speed AB into two parts, Ac and Bc, which relate to one another inversely as the masses B and A, and imagines that A is moved with speed Ac in absolute space, while B is moved with speed Bc in the opposite direction, *together with the relative space*; thus the two motions are equal and opposite to one another, and, since they mutually cancel one another, the two bodies place themselves relative to one another, that is, in absolute space, at rest.⁵⁰ But now B was in motion, *together with the relative space*, with speed Bc, in the direction BA exactly opposite to that of body A, namely, AB. Hence, if the motion of body B is canceled through the impact, the motion of the relative space is not thereby canceled as well. Therefore, *the relative space* moves after the impact, with respect to the two bodies A and B (now at rest in absolute space), in the direction BA with speed Bc, or, equivalently, both bodies move after the impact, with equal speed Bd = Bc, in the direction of the impacting body AB. But now, according to the preceding, the quantity of motion of body B in the direction and with the speed Bc, and hence also that in the direction Bd with the same speed, is equal to the quantity of motion of body A, with the speed and in the direction Ac. Therefore, the action or effect, that is, the motion Bd in relative space that body B receives through the impact, and thus also the action of body A with speed Ac, is always equal to the reaction Bc. Precisely the same law (as mathematical mechanics teaches) holds without modification, if, instead of an impact on a body at rest, one assumes an impact of the same body on one similarly moved. Moreover, the communication of motion through *impact* differs from that through *traction* only in the direction in which the matters resist one another in their motions. It follows, then, that *in all communication of motion* action and reaction are always equal to one another (that every impact can communicate the motion of one body to another only by means of an equal counterimpact, every pressure by

⁵⁰ This is the ideal case of *perfectly inelastic* impact. In the contrary ideal case of *perfectly elastic* impact the two bodies are reflected on impact. According to the Leibnizean principle of the conservation of *vis viva* (cf. note 41 above), only perfectly elastic impact is possible. Kant's view, by contrast, appears to be that perfectly inelastic impact is the "natural case," which one can describe a priori by *abstracting* from all elasticity. See especially Kant's footnote to the following Remark 1 [549], where he claims, in particular, that absolutely hard bodies (since they are by hypothesis perfectly inelastic) could not obey the law of perfectly elastic impact. Compare also the precritical *Neuer Lehrbegriff der Bewegung und Ruhe* (1758) and R. 42 (around 1773) at Ak 14:202–03 (together with Adickes's notes thereto).

means of an equal counterpressure, and every traction only through an equal countertraction).*

[548] *Note 1*

From this there follows a law of nature that is not unimportant for general mechanics: any body, however great its mass, must be *movable* by the impact of any other body, however small its mass or speed. For to the motion of A in the direction AB there necessarily corresponds an opposite and equal motion of B in the direction BA. The two motions cancel one another through their impact in absolute space. But the two bodies thereby acquire a speed $Bd = Bc$ in the direction of the impacting body; hence body B is movable by any force of impact, however small.

Note 2

This, then, is the *mechanical law* of the equality of action and reaction, which rests on the fact that no *communication* of motion takes place, except insofar as we presuppose a *community* of these motions, and thus on the

[547] * In phoronomy, where the motion of a body was considered merely with respect to space, as change of relation *in space*, it was all the same whether I wanted to grant motion to the body in space, or an equal but opposite motion to the relative space instead; the two yielded entirely the same appearance. The quantity of motion of the space was merely the speed, and hence that of the body was likewise nothing but its speed (for which reason it could be considered as a mere movable point). But in mechanics, where a body is considered in motion relative to another, with regard to which, through its motion, it has a *causal relation* – namely, that of moving the body itself – in that it enters into community with [the body] either in its approach through the force of impenetrability or in its withdrawal through that of attraction, it is no longer the same whether I wish to ascribe a motion to one of these bodies, or an opposite motion to the space. For another concept of the quantity of motion now comes into play, namely, not that which is thought merely with respect to space, and consists only in the speed, but rather that whereby the quantity of substance (as moving cause) must be brought into the calculation at the same time; and here it is no longer arbitrary, but rather *necessary*, to assume each of the two bodies as moved, and, indeed, with equal quantity of motion in the opposite direction – but if one is relatively at rest with respect to the space, to ascribe the required motion to it, *together with the space*. For one body cannot act on the other through its own inherent motion, except either in approach by means of repulsive force, or in withdrawal by means of attraction. Now since both forces always act mutually and equally in opposite directions, no body can act by means of them on another body through its motion, without just as much reaction from the other with the same quantity of motion. Hence no body can impart motion to an *absolutely resting* body through its motion; rather, the latter must be moved precisely with the same quantity of motion (together with the space) in the opposite direction as that which it is supposed to receive through the motion, and in the direction, of the first. – The reader will easily become aware that, despite the somewhat unaccustomed character of this mode of representing the communication of motion, it can nonetheless be set in the clearest light, if one does not shy away from the prolixity of the explanation.

fact that no body impacts another that is at rest *relative to it*; rather, the second body is at rest relative to space, only insofar as it moves, *together with this space*, in the same amount, but in the opposite direction, with that motion which then falls to the first as its relative share, and together would originally yield the quantity of motion that we would ascribe to the first in absolute space. For no *motion* that is to be *moving* with respect to another body, can be *absolute*; but if it is relative with respect to the latter, then there is no relation in space that would not be mutual and equal. – There is, however, another law of the equality of action and reaction among matters – namely, a *dynamical law* – not insofar as one matter *communicates* its motion to another, but rather as it *imparts* this motion originally to it, and, at the same time, produces the same in itself through the latter's resistance. This can easily be shown in a similar way. For, if matter A exerts traction on^k matter B, then it *compels* the latter to *approach* it, or, equivalently, it *resists* the force with which the latter might strive to *remove* itself. But since it is all the same whether B removes itself from A, or A from B, this resistance is, at the same time, a resistance exerted by body B against A, insofar as the latter may be striving to remove itself from the former; and so traction and countertraction are equal to one another. In just the same way, if A repels matter B, then A resists [549] the *approach* of B. But, since it is all the same whether B approaches A, or A approaches B, B also resists the approach of A to precisely the same extent; so pressure and counterpressure are also always equal to one another.

Remark 1

Such, then, is the construction of the communication of motion, which at the same time brings with it, as its necessary condition, the law of the equality of action and reaction. *Newton* by no means dared to prove this law a priori, and therefore appealed rather to *experience*. Others, for its sake, introduced into natural science a special force of matter, under the name, first introduced by *Kepler*, of a *force of inertia* (*vis inertiae*),⁵¹ and thus they,

^k zieht.

⁵¹ For *Kepler*, the force of inertia is exerted by a body to preserve its natural state of *rest*, whereas for *Newton*, of course, inertia maintains a body in its natural state of uniform rectilinear motion (which can also be zero). *Newton's* discussion of *vis inertiae* or *vis insita* is in Definition 3 of the *Principia* (Cohen and Whitman, pp. 404–05).

too, derived it in principle from experience. Finally, still others posited in the concept a mere communication of motion, which they viewed as a gradual transfer of the motion of the one body into the other, whereby the mover would have to lose precisely as much motion as it imparts to the moved, until it impresses no more on the latter (that is, until it has already arrived at equality of speed with the latter in the same direction).[†] In this way, they eliminated in principle all reaction, that is, all actual reacting force of the impacted body on the impacting one (which would be capable, for example, of tensing a spring). Moreover, they do not demonstrate what is properly meant in the law in question, and did not at all explain the *communication* of motion itself with regard to its possibility. For the term *transfer* of motion from one body to another explains nothing, and, if it is not meant to be taken literally (in violation of the principle that accidents do not wander from one substance to another),⁵² as if motion were poured from one body into another like water from one glass into another, then we here have precisely the problem of how to make this possibility conceivable – where the explanation thereof in fact rests on precisely the same ground as that from which the law of the equality of action and reaction is derived. One cannot think at all how the motion of a body A must be necessarily combined with the motion of another body B, except by thinking forces in both that pertain to them (dynamically) prior to all motion (for example, repulsion), and now being able to demonstrate that the motion of body A, in its approach towards B, is necessarily combined with the approach of B towards A (and, if B is viewed as at rest, with its motion, *together with its space*, towards A), insofar as the

[549] [†] The equality of action with the (in this case falsely so-called) reaction comes out equally well, if, on the hypothesis of *transfusion* of motion from one body into another, one allows the moved body A to deliver its entire motion in an instant to the body at rest, so that it is itself at rest after the impact – this case was inevitable as soon as one thought both bodies as *absolutely hard* (which property must be distinguished from elasticity). But since this law of motion would agree neither with experience, nor with itself in application, no other remedy was known but to deny the existence of absolutely hard bodies, which amounted to admitting the contingency of this law, in that it was supposed to rest on a particular quality of matters that move one another. In our presentation of this law, by contrast, it is all the same whether one wishes to think the colliding bodies as absolutely hard or not. It is completely inconceivable to me, however, how the *transfusionists* of motion would explain, in their fashion, the motion of *elastic* bodies through impact. For here it is clear that the resting body does not, merely as resting, acquire motion lost by the impacting body, but that, in the collision, it exerts actual force on the latter in the opposite direction, so as to compress, as it were, a *spring* between the two, which requires just as much actual motion on its part (but in the opposite direction) as the moving body itself has need of for this purpose.

⁵² Kant gives the principle in Latin: *accidentia non migrant e substantiis in substantias*.

bodies, with their (original) moving forces, are considered to be in motion merely relative to one another. This latter can be comprehended completely a priori, once it is seen that, whether body B is at rest or moved relative to the empirically knowable space, it still must be viewed, relative to body A, as necessarily moved, and, indeed, in the opposite direction. For otherwise no influence of B on the repulsive force of the two bodies would occur, and without this no mechanical action of the matters on one another – that is, no communication of motion by impact – is possible at all.

Remark 2

Regardless of the famous name of its creator, the terminology of inertial force (vis inertiae) must therefore be entirely banished from natural science, not only because it carries with it a contradiction in terms, nor even because the law of inertia (lifelessness) might thereby easily be confused with the law of reaction in every communicated motion, but primarily because the mistaken idea of those who are not properly acquainted with the mechanical laws is thereby maintained and even strengthened – according to which the reaction of bodies discussed under the name of inertial force would amount to a draining off, diminution, or eradication of the motion in the world; but the mere communication of motion would not be effected thereby, because the moving body would have to apply a part of its motion solely in overcoming the inertia of the one at rest (which would then be a pure loss), and could only set the latter in motion with the remaining part; but if none were left over, it would completely fail to move the latter by its impact, because of its great mass. But nothing can resist a motion except the opposite motion of another body, and certainly not its state of rest. Thus here the inertia of matter, that is, the mere incapacity for self-movement, is not the cause of a resistance. A special, entirely peculiar force merely to resist, without being able to move a body, under the name of an inertial force, would be a word without any meaning. The three laws of general mechanics could therefore more appropriately be named the laws of *self subsistence*, *inertia*, and *reaction of matters* (lex subsistentiae, inertiae, et antagonismi) *in all of their changes*. That these laws, and thus all Propositions of the present science, precisely answer to the categories of *substance*, *causality*, and *community*, insofar as these concepts are applied to matter, needs no further discussion. [551]

General Remark to Mechanics

The communication of motion occurs only by means of such moving forces as also inhere in a matter at rest (impenetrability and attraction). The action of a moving force on a body in an instant is its *solicitation*; the speed effected in the latter through solicitation, insofar as it can increase in equal proportion to the time, is the *moment* of acceleration.¹ (The moment of acceleration must therefore contain only an infinitely small speed, because otherwise the body would thereby attain an infinite speed in a given time, which is impossible; moreover, the possibility of *acceleration*^m in general, by means of a continued moment thereof, rests on the law of inertia.) The solicitation of matter by expansive force (of compressed air bearing a weight, for example) occurs always with a finite speed, but the speed thereby impressed on (or extracted from) another body can only be infinitely small; for expansion is only a surface force, or, what is the same, the motion of an infinitely small quantum of matter, which therefore must occur with finite speed, in order to be equal to the motion of a body with finite mass and infinitely small speed (a weight). By contrast, attraction is a penetrating force, and with such a force a finite quantum of matter exerts moving force on another similarly finite quantum. The solicitation [552] of attraction must therefore be infinitely small, because it is equal to the moment of acceleration (which must always be infinitely small) – which is not the case with repulsion, since an infinitely small part of matter is to impress a moment on a finite one. No attraction can be thought with a finite speed, without the matter having to *penetrate* itself by its own force of attraction. For the attraction that a finite quantity of matter exerts on a finite one, with a finite speed, must at all points of the compression be greater than any finite speed whereby the matter reacts through its impenetrability, but with only an infinitely small part of the quantity of its matter. If the attraction is only a surface force, as we think cohesion to be, then the opposite of this would result. But it is impossible to think cohesion in this way if it is to be true attraction (and not mere external compression).

An absolutely hard body would be one whose parts attractedⁿ one another so strongly that they could neither be separated, nor changed *in their situation* relative to one another, by any weight. Now since the parts of the matter of such a body would have to attract^o one another with

¹ *Moment der Acceleration.*

^m *Beschleunigung.*

ⁿ *zögen.*

^o *ziehen.*

a moment of acceleration that would be infinite with respect to that of gravity, but finite with respect to that of the mass that is to be driven thereby, the resistance by means of impenetrability, as expansive force, since it always occurs with an infinitely small quantity of matter, would then have to take place with a more than finite speed of solicitation, that is, the matter would strive to expand with infinite speed, which is impossible. Hence an absolutely hard body, that is, one that would, on impact, *instantaneously* oppose a body moved at finite speed, with a resistance equal to the total force of that body, is impossible.⁵³ Consequently, by means of its impenetrability or cohesion, a matter attains instantaneously only an infinitely small resistance to the force of a body in finite motion. And from this there now follows the mechanical law of continuity (lex continui mechanica): namely, that in no body is the state of rest or motion, or the speed or direction of the latter, changed by impact instantaneously, but only in a certain time, through an infinite series of intermediate states, whose difference from one another is less than that between the first state and the last. A moved body that impacts on a matter is thus not brought into a state of rest by the latter's resistance all at once, but only through a continuous retardation; and one that was at rest is put into motion only through a continuous acceleration;^p and it is changed from one degree of speed to another only in accordance with the same rule. In the same way, the direction of its motion is changed into one that makes an angle with it no otherwise than by means of all possible intermediate directions, that is, by means of motion in a curved line. (And, on similar grounds, this [553] law can be extended also to the change of state of a body by attraction.) This lex continui is based on the law of the inertia of matter, whereas the *metaphysical* law of continuity would have to be extended to all changes in general (inner as well as outer), and thus would have to be based on the mere *concept of a change in general*, as quantity, and on the generation thereof (which would necessarily proceed continuously in a certain time, as does time itself). This metaphysical law can therefore find no place here.⁵⁴

^p *Acceleration.*

⁵³ Compare note 50 above.

⁵⁴ See A206–11/B252–56; and compare A171–72/B212–13.

Fourth Chapter

Metaphysical Foundations of Phenomenology

Explication

Matter is the movable insofar as it, as such a thing, can be an object of experience.

Remark

Motion, like everything that is represented through the senses, is given only as appearance. For its representation to become experience, we require, in addition, that something be thought through the understanding—namely, besides the mode in which the representation inheres in the *subject*, also the determination of an *object* thereby. Hence the movable, as such a thing, becomes an object of experience, when a certain *object* (here a material thing) is thought as *determined* with respect to the *predicate* of motion. But motion is change of relation in space. There are thus always two correlates here, such that either, *first*, the change can be attributed in the appearance to one just as well as to the other, and either the *one* or the *other* can be said to be moved, because the two cases are equivalent;^a or, *second*, one must be thought in experience as moved to the exclusion of the other; or, *third*, both must be necessarily represented through reason as equally moved. In the appearance, which contains nothing but the relation

[555] in the motion (with respect to its change), none of these determinations are contained. But if the movable, *as such a thing*, namely, with respect

^a *weil beides gleichgültig ist.*

to its motion, is to be thought of as determined for the sake of a possible experience, it is necessary to indicate the conditions under which the object (matter) must be determined in one way or another by the predicate of motion. At issue here is not the transformation of semblance^b into truth, but of appearance into experience; for, in the case of semblance, the understanding with its object-determining judgments is always in play, although it is in danger of taking the subjective for objective; in the appearance, however, no judgment of the understanding is to be met with at all⁵⁵ – which needs to be noted, not merely here, but in the whole of philosophy, because otherwise, when appearances are in question, and this term is taken to have the same meaning as semblance, one is always poorly understood.

Proposition 1

The rectilinear motion of a matter with respect to an empirical space, as distinct from the opposite motion of the space, is a merely *possible* predicate. The same when thought in no relation at all to a matter external to it, that is, *as absolute motion*, is *impossible*.

Proof

Whether a body is said to be moved in a relative space, and the latter at rest, or whether, conversely, the latter shall be said to be moved, with the same speed in the opposite direction, with the former at rest, is not a dispute about what pertains to the object, but only about its relation to the subject, and belongs therefore to appearance and not experience. For if the observer locates himself in that space as at rest, the body counts as moved for him; if he locates himself (at least in thought) in another

^b *Schein*.

⁵⁵ Compare the discussion in Remark III to the first part of the *Prolegomena* (Ak 4:291): “The senses represent to us the movement of the planets as now progressive, now retrogressive; and herein is neither falsehood nor truth, because, so long as one acquiesces for the moment in this being only appearance, one does not yet judge in any way concerning the objective character of the motion. However, because a false judgment can easily arise if the understanding has not taken sufficient care to prevent the subjective mode of representation from being taken for objective, one says that they present a semblance [*sie scheinen*] of retrogression. Yet the semblance [*Schein*] here is not to be charged to the senses but to the understanding, whose province alone it is to make an objective judgment from the appearances.”

space comprehending the first, relative to which the body is likewise at rest, then that relative space counts as moved. Thus in experience (a cognition that determines the object validly for all appearances) there is no difference at all between the motion of the body in the relative space, and the body being at rest in absolute space, together with an equal and opposite motion of the relative space. Now the representation of an object [556] through one of two predicates, which are equally valid with respect to the object, and differ from one another only in regard to the subject and its mode of representation, is not a determination in accordance with a *disjunctive judgment*, but merely a choice in accordance with an *alternative judgment*. (In the former, of two *objectively* opposed predicates, one is assumed to the exclusion of the other for the determination of the object; in the latter, of two judgments objectively equivalent, yet subjectively opposed to one another, one is assumed for the determination of the object without excluding its opposite – and thus by mere choice.)* This means that through the concept of motion, as object of experience, it is in itself undetermined, and therefore equivalent, whether a body be represented as moved in the relative space, or the latter with respect to the former. Now that which is in itself undetermined with respect to two opposed predicates is to that extent *merely possible*. Hence the rectilinear motion of a matter in empirical space, as distinct from the equal opposite motion of the space, is a merely possible predicate in experience – which was the first [thing to be proved].

Since, moreover, a relation, and thus also a change thereof, that is, motion, can be an object of experience only insofar as both correlates are objects of experience, whereas the pure space that is also called absolute space, in contrast to relative (empirical) space, is no object of experience, and in general^c is nothing, a rectilinear motion without reference to anything empirical, that is, absolute motion, is completely impossible – which was the second [thing to be proved].

Remark

This Proposition determines the modality of motion with respect to *phoronomy*.

* More will be said about this distinction between disjunctive and alternative opposition in the General Remark to this chapter.

^c *überall*.

Proposition 2

The circular motion of a matter, as distinct from the opposite motion of the space, is an *actual* predicate of this matter; by contrast, the opposite motion of a relative space, assumed instead of the motion of the body, is [557] no actual motion of the latter, but, if taken to be such, is mere semblance.

Proof

Circular motion (like all curvilinear motion) is a continuous change of rectilinear motion, and, since the latter is itself a continuous change of relation with respect to the external space, circular motion is a change of a change in these external relations in space, and is thus a continuous arising of new motions. Now since, according to the law of inertia, a motion, in so far as it arises, must have an external cause, while the body, at every point on this circle (according to precisely the same law), is striving, for its own part, to proceed in the straight line tangent to the circle, which motion acts in opposition to this external cause, it follows that every body in circular motion manifests,^d by its motion, a moving force. But the motion of the space, as distinct from that of the body, is merely *phoronomic*, and has no moving force. Thus the judgment that here either the body is moved, or the space is moved in the opposite direction, is a *disjunctive* judgment, whereby, if one of the terms (namely, the motion of the body) is posited, the other, (namely, that of the space) is excluded. Thus the circular motion of a body, as distinct from that of the space, is an *actual* motion, so that the latter, even though it agrees with the former according to the appearance, nevertheless contradicts it in the context of all appearances, that is, of a possible experience, and so is nothing but mere semblance.

Remark

This Proposition determines the modality of motion with respect to *dynamics*; for a motion that cannot take place without the influence of a continuously acting external moving force manifests,^e directly or indirectly, originally moving forces of matter, whether of attraction or

^d beweiset. ^e beweiset.

repulsion. – Moreover, *Newton's* Scholium to the Definitions he has prefixed to his *Principia* may be consulted on this subject, towards the end, where it becomes clear that the circular motion of two bodies around a
[558] common central point (and thus also the axial rotation of the earth) can still be known by experience even in empty space, and thus without any empirically possible comparison *with an external space*;⁵⁶ so that a motion, therefore, which is a change of external relations in space, can be empirically given, even though this space is not itself empirically given, and is no object of experience. This is a paradox that deserves to be solved.

Proposition 3

In every motion of a body, whereby it is moving relative to another, an opposite and equal motion of the latter is *necessary*.

Proof

According to the Third Law of Mechanics (Proposition 4), the communication of motion of bodies is possible only by the community of their original moving forces, and the latter only by mutually opposite and equal motion. The motion of both is therefore actual. But since the actuality of this motion does not rest (as in the second Proposition) on the influence of external forces, but follows immediately and unavoidably from the concept of the relation of the *moved* in space to anything else *movable* thereby, the motion of the latter is *necessary*.

Remark

This Proposition determines the modality of motion with respect to mechanics. – Moreover, it is obvious that these three Propositions determine the motion of matter with respect to its *possibility*, *actuality*, and *necessity*, and thus with respect to all three categories of *modality*.

⁵⁶ Compare Kant's second footnote to the General Remark to Phenomenology [562], and see note 58 below.

General Remark to Phenomenology

Thus here appear three concepts, whose use in general natural science is unavoidable, and whose precise determination is therefore necessary, although not that easy or comprehensible – namely, [first,] the concept of *motion in relative (movable) space*, second, that of *motion in absolute* [559] *(immovable) space*, and third, that of *relative motion* in general, as distinct from absolute motion. The concept of absolute space is the basis for all of them. But how do we arrive at this peculiar concept, and what underlies the necessity of its use?

It cannot be an object of experience, for space without matter is no object of perception, and yet it is a necessary concept of reason, and thus nothing more than a mere *idea*. For in order that motion may be given, even merely as appearance, an empirical representation of space is required, with respect to which the movable is to change its relation; but the space that is to be perceived must be material, and thus itself movable, in accordance with the concept of a matter in general. Now, to think of it as moved, one may think it only as contained in a space of greater extent, and take the latter to be at rest. But the same can be done with the latter, with respect to a still further extended space, and so on to infinity, without ever arriving by experience at an immovable (immaterial) space, with respect to which either motion or rest might absolutely^f be attributed to any matter. Rather, the concept of these relational determinations will have to be continually revised, according to the way that we will consider the movable in relation to one or another of these spaces. Now since the condition for regarding something as at rest or moved is always conditioned in turn, *ad infinitum*, in relative space, it becomes clear, *first*, that all motion or rest can be relative only and never absolute, that is, that matter can be thought as moved or at rest solely in relation to matter, and never with respect to mere space without matter, so that absolute motion, thought without any relation of one matter to another, is completely impossible; and *second*, for precisely this reason, that no concept of motion or rest valid *for all appearance* is possible in relative space. Rather, one must think a space in which the latter can itself be thought as moved, but which depends for its determination on no further empirical space, and thus is not conditioned in turn – that is, an absolute space to which all relative motions can be

^f *schlechtin*.

referred, in which everything empirical is movable, precisely so that in it all motion of material things^g may count as merely relative with respect to one another, as alternatively mutual,[†] but none as absolute motion or rest [560] (where, while one is said to be moved, the other, in relation to which it is moved, is nonetheless represented as absolutely^h at rest). Absolute space is therefore necessary, not as a concept of an actual object, but rather as an idea, which is to serve as a rule for considering all motion therein merely as relative; and all motion and rest must be reduced to absolute space, if the appearance thereof is to be transformed into a determinate concept of experience (which unites all appearances).

Thus the rectilinear motion of a body in relative space is reduced to absolute space, when I think the body as in itself at rest, but this space as moved in the opposite direction in absolute space (which is not apprehended by the senses), and when I think this representation as that which yields precisely the same appearance, whereby all possible appearances of rectilinear motions that a body may have at the same time are reduced to the concept of experience which unites them all, namely, that of merely relative motion and rest.

Because *circular motion*, according to the second Proposition, can be given as actual motion in experience, even without reference to the external empirically given space, it indeed seems to be absolute motion. For relative motion with respect to the external space (for example, the axial rotation of the earth relative to the stars of the heavens) is an *appearance*, in place of which the opposite motion of this space (of the heavens) in the same time can be supposed as completely equivalent to the former;

[559] [†] In logic the *either—or* always signifies a *disjunctive* judgment, where, if the one is true, the other must be false. For example, a body is *either* moved *or* not moved, that is, at rest. For here [in logic] one speaks solely of the relation of the cognition to the object. In the doctrine of appearance, where it is a matter of the relation to the subject, so as to determine the relation to the object therefrom, the situation is different. For here the proposition that the body is either moved and the space at rest, or conversely, is not a disjunctive proposition in an objective relation, but only in a subjective one, and the two judgments contained therein are valid *alternatively*. In precisely the same phenomenology, where the motion is considered, not merely phoronomically, but rather dynamically, the disjunctive proposition is instead to be taken in an *objective* meaning; that is, I cannot assume, in place of the rotation of a body, a state of rest of the latter and the opposite motion of the space instead. But wherever the motion is considered *mechanically* (as when a body approaches another seemingly at rest), then the formally disjunctive judgment must be used *distributively* with respect to the object, so that the motion must not be attributed *either* to one *or* the other, but rather an equal share of it to each. This distinction among *alternative*, *disjunctive*, and *distributive* determinations of a concept with respect to opposing predicates has its importance, but cannot be further discussed here.

^g *Materiellen.* ^h *schlechtin.*

but, according to the Proposition in question, it certainly may not be so substituted in experience. Hence, that rotation is not to be represented as externally relative, which sounds as if this kind of motion is to be taken as absolute. [561]

But it should be noted that it is here a question of true (actual) *motion*, which does not, however, *appear* as such, so that, if one wished to evaluate it merely in accordance with empirical relations to space, it could be taken for *rest*; it is a question, that is, of *true motion* as distinct from *semblance*, but not of absolute motion in contrast to relative. Thus, circular motion, although it in fact exhibits no change of place in the appearance, that is, no phoronomic change in the relations of the moved body *to* (empirical) *space*, exhibits nonetheless a continuous dynamical change, demonstrable through experience, in the relations of matter *within its space*, for example, a continual diminution of attraction in virtue of a striving to escape, as an action or effectⁱ of the circular motion, and thereby assuredly indicates its difference from semblance. For example, one may represent to oneself the earth as rotating on its axis in infinite empty space, and also verify this motion by experience, even though neither the relation of the earth's parts among one another, nor to the space outside it, is changed phoronomically, that is, in the appearance. For, with respect to the first, as empirical space, nothing changes its position in or on the earth; and, as regards the second, which is completely empty, no externally changed relation, and thus no appearance of a motion, can take place anywhere. But if I represent to myself a deep hole descending to the center of the earth, and I let a stone fall into it, I find, however, that the falling stone deviates from its perpendicular direction continuously, and, in fact, from west to east, even though gravity, at all distances from the center of the earth, is always directed towards it, and I conclude, therefore, that the earth is rotating on its axis from west to east.^j Or, if I also remove the stone further out from the surface of the earth, and it does not remain over the same point of the surface, but moves away from it from east to west,⁵⁷ then I will infer

ⁱ *Wirkung* (action or effect). ^j *von Abend gegen Morgan*.

⁵⁷ In the original edition: *von Osten nach Westen*. The Akademie edition substitutes *von Westen nach Osten* here, on the grounds that the experiment of *throwing* a stone outward from the surface of the earth so as to observe its deviation from east to west is much more difficult than the experiment of *dropping* a stone from a tower so as to observe its deviation from west to east. In any case, however, what Kant is describing here – just as in the case of dropping a stone into a deep hole directed toward the earth's center – is the action of the *Coriolis force* of the earth's rotation.

to the very same previously mentioned axial rotation of the earth, and both observations will be sufficient to prove the actuality of this motion. The change of relation to the external space (the starry heavens) does not suffice for this, since it is mere appearance, which may proceed from two in fact opposing grounds, and is not a cognition derived from the explanatory ground of all appearances of this change, that is, experience. But that this motion, even though it is no change of relation to the empirical space, is nevertheless not absolute motion, but rather a continuous change in the relations of matters to one another, which, although represented in absolute space, is thus actually only relative, and, for just that reason, is true motion – this rests on the representation of the mutual and continuous *withdrawal* of any part of the earth (outside the axis) from any other part [562] lying diametrically opposite to it at the same distance from the center. For this motion is actual in absolute space, in that the reduction of the distance in question, which gravity by itself would induce in the body, is thereby continuously made up, and, in fact, without any dynamical repulsive cause (as may be seen from the example chosen by Newton in the *Principia*, page 10 of the 1714 edition[‡]); hence it is made up through actual motion, which relates, however, to the space inside of the moved matter (namely, its center), and not to that outside it.⁵⁸

In the case of the *third Proposition*, to show the truth of the mutually opposed and equal motions of the two bodies, even without reference to the empirical space, we do not even need the active dynamical influences, given through experience, that are required in the second case (gravity or a

[‡] There he says: “It is certainly very difficult to find out the true motions of individual bodies and actually to differentiate them from apparent motions, because the parts of that immovable space in which the bodies truly move make no impression on our senses. Nevertheless, the case is not utterly hopeless.” He then lets two spheres connected by a cord revolve around their common center of gravity in empty space, and shows how the actuality of their motion, together with its direction, can nonetheless be discovered by means of experience. I have attempted to show this also in the case of the earth moved around its axis, in somewhat altered circumstances.

⁵⁸ *Denn diese Bewegung ist im absoluten Raume wirklich, indem dadurch der Abgang der gedachten Entfernung, den die Schwere für sich allein dem Körper zuziehen würde, und zwar ohne alle dynamische zurücktreibende Ursache (wie man aus dem von Newton Prin. Ph. N. pag. 10 Edit. 1714* gewählten Beispiele ersehen kann), mithin durch wirkliche, aber auf den innerhalb der bewegten Materie (nämlich das Centrum derselben) beschlossenen, nicht aber auf den äußeren Raum bezogene Bewegung continuirlich ersetzt wird.* (Here the date of the second edition of the *Principia* should, rather, be 1713.) In the footnote Kant quotes from the Latin: *Motus quidem veros corporum singulorum cognoscere et ab apparentibus actu discriminare difficillimum est: propterea, quod partes spatii illius immobilis, in quo corpora vere moventur, non incurrunt in sensus. Causa tamen non est prorsus desperata.* The quotation given is from Cohen and Whitman, p. 414.

tensed cord). Rather, the mere dynamical possibility of such an influence, as a property of matter (repulsion or attraction), leads by itself, and from mere concepts of a relative motion, from the motion of one body to the simultaneous equal and opposite motion of the other, when the latter is considered in absolute space, that is, in accordance with truth. Hence like everything sufficiently provable from mere concepts, this is a law of an absolutely necessary counter-motion.

There is thus no absolute motion, even when a body in empty space is thought as moved with respect to another; their motion here is not considered relative to the space surrounding them, but only to the space between them, which, considered as absolute space, alone determines their external relations to one another, and is in turn only relative. Absolute motion would thus be only that which pertained to a body without relation to any other matter. Only the rectilinear motion of the *cosmos*,^k that is, the system of all matter, would be such a motion. For if, outside a matter, there were any other at all, even separated from it by empty space, then the motion would already be relative. For this reason, any proof of a law of motion, which amounts to showing that its opposite would have to result in a rectilinear motion of the entire cosmic system,^l is an apodictic proof of its truth, simply because absolute motion would then follow, which is [563] utterly impossible. Such is the law of *antagonism* in all community of matter through motion. For any deviation from it would shift the common center of gravity of all matter, and thus the entire cosmic system, from its place – which would not happen, however, if one wanted to imagine this system as rotating on its axis. Hence it is always possible to think such a motion, although to suppose it would, so far as one can see, be entirely without any conceivable use.

To the various concepts of motion and moving forces there also correspond the various concepts of *empty space*. Empty space in the *phoronomical* sense, which is also called absolute space, should not properly be called an empty space; for it is only the idea of a space, in which I abstract from all particular matter that makes it an object of experience, in order to think therein the material space, or any other empirical space, as movable, and thereby to think of motion, not merely in a one-sided fashion as absolute, but always mutually, as a merely relative predicate. It is therefore nothing at all that belongs to the existence of things, but

^k *Weltganzen*. ^l *Weltgebäude*.

merely to the determination of concepts, and to this extent no empty space *exists*. Empty space in the *dynamical* sense is that which is not filled, that is, in which no other movable resists the penetration of a movable, and thus no repulsive force acts; it can either be empty space *within* the world (vacuum mundanum) or, if the latter is represented as bounded, empty space *outside* the world (vacuum extramundanum). The former, too, can be represented either as dispersed (vacuum disseminatum, which constitutes only a part of the volume of matter), or as accumulated empty space (vacuum coacervatum, which separates bodies, for example, the heavenly bodies, from one another). This latter distinction is certainly not an essential one, since it rests only on a difference in the locations assigned to empty space within the world, but is still employed for various purposes: the first, to derive specific differences in density, and the second, to derive the possibility of a motion in the universe free from all external resistance. That it is not *necessary* to assume empty space for the *first purpose* has already been shown in the General Remark to Dynamics; but that it is *impossible* can in no way be proved from its concept alone, in accordance with the principle of noncontradiction. Nevertheless, even if no merely logical reason for rejecting this kind of empty space were to be found here, there could still be a more general physical reason for expelling it from the doctrine of nature – that of the possibility of the composition of a matter in general, if only this were better understood. For if the *attraction* assumed in order to explain the cohesion of matter should

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be only apparent, not true attraction, and were merely the effect, say, of a *compression* by external matter (the aether) distributed everywhere in the universe, which is itself brought to this pressure only through a universal and original attraction, namely, gravitation (a view that is supported by several reasons), then empty space within matter, although not logically impossible, would still be so dynamically, and thus physically, since any matter would expand of itself into the empty spaces assumed within it (since nothing resists its expansive force here), and would always keep them filled. An empty space *outside the world*, understood as the totality of preeminently attractive matters (the large heavenly bodies), would be impossible for precisely the same reasons, since in accordance with their mass, as the distance from them increases, the attractive force on the aether (which encloses all these bodies, and, driven by that force, conserves them in their density by compression) decreases in inverse proportion, so that the latter would itself only decrease indefinitely in density, but nowhere

leave space completely empty. It should not surprise anyone, however, that this refutation of empty space proceeds entirely hypothetically, for the assertion of empty space fares no better. Those who venture to settle this disputed question dogmatically, whether for or against, rely in the end on plainly metaphysical presuppositions, as can be seen from the Dynamics – and here it was necessary at least to show that these can do nothing at all to resolve the problem. As for empty space in the third, or *mechanical sense*, it is the emptiness accumulated within the cosmos to provide the heavenly bodies with free motion. It is easy to see that the possibility or impossibility of this does not rest on metaphysical grounds, but on the mystery of nature, difficult to unravel, as to how matter sets limits to its own expansive force. Nevertheless, if one grants what was said in the General Remark to Dynamics concerning the possibility of an ever-increasing expansion of specifically different materials, at the same quantity of matter (in accordance with their weight), it may well be unnecessary to suppose an empty space for the free and enduring motion of the heavenly bodies; since even in spaces completely filled, the resistance can still be thought as small as one likes.

And so ends the metaphysical doctrine of body with the *empty*, and therefore the inconceivable, wherein it shares the same fate as all other attempts of reason, when it strives after the first grounds of things in a retreat to principles – where, since its very nature entails that it can never conceive anything, except in so far as it is determined under given conditions, and since it can therefore neither come to a halt at the conditioned, nor make the unconditioned comprehensible, nothing is left to it, when thirst for knowledge invites it to comprehend the absolute totality of all conditions, but to turn away from the objects to itself, so as to explore and determine, not the ultimate limits of things, but rather the ultimate limits of its own unaided powers. [565]

Glossary

German-English

Abwiegen	<i>weighing</i>	
Acceleration	<i>acceleration</i>	(cf. Beschleunigung)
Äther	<i>aether</i>	
allgemeine Anziehung	<i>universal attraction</i>	(cf. Gravitation)
Anziehung	<i>attraction</i>	
Anziehungskraft	<i>attractive force</i>	
Auflösung	<i>analysis</i>	
Auflösungsmittel	<i>solvent</i>	
aufzulösende Materie	<i>solute</i>	
Ausdehnung	<i>extension</i>	
Ausdehnungsvermögen	<i>expansive capacity</i>	
ausspannende Kraft	<i>expansive force</i>	
Bebung	<i>tremor, vibration</i>	(cf. Zitterung)
beharrlich	<i>perduring</i>	
Beobachtung	<i>observation</i>	
Berührung	<i>contact</i>	
Beschaffenheit	<i>constitution</i>	
Beschleunigung	<i>acceleration</i>	(cf. Acceleration)
Bestimmungsgrund	<i>ground of determination</i>	
Bestrebung	<i>striving</i>	
bewegende Kraft	<i>moving force</i>	
Beweglichkeit	<i>movability</i>	
Bewegung	<i>motion</i>	
Bewegungslehre	<i>doctrine of motion</i>	
blasericht	<i>vesicular</i>	
chemische Verwandtschaft	<i>chemical affinity</i>	

Chymie	<i>chemistry</i>	
Construction	<i>construction</i>	
construieren	<i>construct</i>	
darstellen	<i>present</i>	
dauern	<i>endure</i>	
daurende Gegenwart	<i>enduring presence</i>	
Diagonallinie	<i>diagonal</i>	
Dichtigkeit	<i>density</i>	
Druck	<i>pressure</i>	
durchdringen	<i>penetrate</i>	(cf. eindringen)
durchdringende Kraft	<i>penetrating force</i>	
Durchdringung	<i>penetration</i>	
eindringen	<i>penetrate into</i>	(cf. durchdringen)
eindrücken	<i>impress</i>	
elastische Materie	<i>elastic material</i>	
Elastizität	<i>elasticity</i>	
Entfernung	<i>distance</i>	
entgegengesetzt	<i>opposite</i>	
entgegenwirken	<i>counteract</i>	(cf. gegenwirken)
erfordern	<i>require</i>	
erkennen	<i>cognize</i>	
Erkenntnis	<i>cognition</i>	
Erkenntnisart	<i>mode of cognition</i>	
Erklärungsart	<i>mode of explanation</i>	
Erscheinung	<i>appearance</i>	
Erscheinungslehre	<i>doctrine of appearance</i>	
Erstarrung	<i>rigidification</i>	
erteilen	<i>impart</i>	(cf. mitteilen)
Experiment	<i>experiment</i>	
Feder	<i>spring</i>	(cf. Springfeder)
fest	<i>solid</i>	
Festigkeit	<i>solidity</i>	(cf. Solidität)
Flächenkraft	<i>surface force</i>	
flüssig	<i>fluid</i>	
fortschreitend	<i>progressive</i>	
fortschreitende Bewegung	<i>progressive motion</i>	
Gärung	<i>fermentation</i>	
gegenwirken	<i>react</i>	(cf. entgegenwirken)
Gegenwirkung	<i>reaction</i>	
Gemeinschaft	<i>community</i>	
Geschwindigkeit	<i>speed</i>	

Gestalt	<i>shape, figure</i>	
gleichartig	<i>homogeneous</i>	
gleichförmig	<i>uniform</i>	
gleichgeltend	<i>equivalent</i>	
gleichgewicht	<i>equilibrium</i>	
gleichmäßig	<i>proportionate</i>	
gleichgültig	<i>indifferent</i>	
gradlinicht	<i>rectilinear</i>	
Grenze	<i>boundary, limit</i>	
Gravitation	<i>gravitation</i>	(cf. allgemeine Anziehung)
Größe	<i>quantity, magnitude</i>	
Grundkraft	<i>fundamental force</i>	
Grundlage	<i>basis</i>	
Grundsatz	<i>principle</i>	
Halbmesser	<i>radius</i>	
Hinzutuuung	<i>addition</i>	
Hydrodynamik	<i>hydrodynamics</i>	
Hydrostatik	<i>hydrostatics</i>	
Hylozoismus	<i>hylozoism</i>	
Hypothese	<i>hypothesis</i>	
Klebrigkeit	<i>viscosity</i>	
Kongruenz	<i>congruence</i>	
Körper	<i>body</i>	
Körperlehre	<i>doctrine of body</i>	
körperlicher Raum	<i>volume</i>	
Körperchen	<i>particle</i>	
Kraft	<i>force</i>	
Kreisbewegung	<i>circular motion</i>	
krummlinicht	<i>curvilinear</i>	
Lage	<i>position, situation</i>	(cf. Ort)
die Leere	<i>void</i>	
löchericht	<i>porous</i>	
Masse	<i>mass, measure</i>	
Menge	<i>aggregate</i>	
mitteilen	<i>communicate</i>	(cf. erteilen)
Mitteilung	<i>communication</i>	
mittelbar	<i>mediate</i>	
Monas	<i>monad</i>	
nähren	<i>approach</i>	
Naturlehre	<i>doctrine of nature</i>	
Naturwissenschaft	<i>natural science</i>	

oscillierend	<i>oscillating</i>	
Ort	<i>place</i>	(cf. Lage)
Raumesinhalt	<i>volume</i>	
Reibung	<i>friction</i>	
repellierende Kraft	<i>repulsive force</i>	(cf. zurückstossende Kraft, Zurückstossungskraft)
reziprozierende Bewegung	<i>reciprocating motion</i>	
Richtung	<i>direction</i>	
Ruhe	<i>rest</i>	
Scheidung	<i>decomposition</i>	(cf. Trennung)
Schein	<i>semblance</i>	
Schnelligkeit	<i>rapidity</i>	
schwankende Bewegung	<i>oscillatory motion</i>	
Schwere	<i>gravity, weight</i>	
Solidität	<i>solidity</i>	(cf. Festigkeit)
Sollizitation	<i>sollicitation</i>	
Sphäre der Wirksamkeit	<i>sphere of activity</i>	
Springfeder	<i>spring</i>	
spröde	<i>brittle</i>	
starr	<i>rigid</i>	
Stetigkeit	<i>continuity</i>	
Stoff	<i>material</i>	
Stoss	<i>impact</i>	
Teilbarkeit	<i>divisibility</i>	
Teilung	<i>division</i>	
Trägheit	<i>inertia</i>	
Transfusion	<i>transfusion</i>	
Trennung	<i>separation</i>	(cf. Scheidung)
Undurchdringlichkeit	<i>impenetrability</i>	
ursprünglich	<i>original</i>	
Veränderung	<i>change</i>	
Verbreitung	<i>diffusion</i>	
vereinigen	<i>unite</i>	
Verminderung	<i>diminution</i>	
Vermögen	<i>power, capacity</i>	
Vernunft	<i>reason</i>	
Vernunftzusammenhang	<i>rational coherence</i>	
Verschiebung	<i>displacement</i>	
Verstand	<i>understanding</i>	
Vielheit	<i>plurality</i>	
Vorstellung	<i>representation</i>	

Vorstellungsart	<i>mode of representation</i>	
Wagschale	<i>scale</i>	
Wärme	<i>heat</i>	
Wärmestoff	<i>caloric</i>	
Wahrnehmung	<i>perception</i>	
Wechselwirkung	<i>interaction</i>	
Weltkörper	<i>heavenly body</i>	
Wesen	<i>essence, being</i>	
Widerstand	<i>resistance</i>	
wirklich	<i>actual</i>	
Wirkung	<i>action, effect</i>	
Wirkung in die Ferne	<i>action at a distance</i>	
Wirkungsraum	<i>space of activity</i>	
zirkulierende Bewegung	<i>circulating motion</i>	
Zitterung	<i>vibration</i>	
Zug	<i>traction</i>	
zurücklegen	<i>traverse</i>	
Zurückstossung	<i>repulsion</i>	
Zurückstossungskraft	<i>repulsive force</i>	(cf. repellierende Kraft)
zurückstossende Kraft	<i>repulsive force</i>	
zurücktreiben	<i>repel</i>	
Zusammendrückung	<i>compression</i>	
Zusammenhang	<i>cohesion</i>	
zusammengesetzte Bewegung	<i>composite motion</i>	
Zusammensetzung	<i>composition</i>	
Zwischenräume	<i>interstices</i>	

English–German

acceleration	<i>Acceleration, Beschleunigung</i>
action (cf. effect)	<i>Wirkung</i>
action at a distance	<i>Wirkung in die Ferne</i>
aether	<i>Äther</i>
aggregate	<i>Menge</i>
analysis	<i>Auflösung</i>
appearance	<i>Erscheinung</i>
approach	<i>nähren</i>
attraction	<i>Anziehung</i>
attractive force	<i>Anziehungskraft</i>
basis	<i>Grundlage</i>

being (cf. essence)	<i>Wesen</i>
body	<i>Körper</i>
brittle	<i>spröde</i>
caloric	<i>Wärmestoff</i>
capacity	<i>Vermögen</i>
chemical affinity	<i>chemische Verwandtschaft</i>
circular motion	<i>Kreisbewegung</i>
circulating motion	<i>zirkulierende Bewegung</i>
cognition	<i>Erkenntnis</i>
cognize	<i>erkennen</i>
cohesion	<i>Zusammenhang</i>
communication	<i>Mitteilung</i>
community	<i>Gemeinschaft</i>
composite motion	<i>zusammengesetzte Bewegung</i>
composition	<i>Zusammensetzung</i>
compression	<i>Zusammendrückung</i>
congruence	<i>Kongruenz</i>
contact	<i>Berührung</i>
continuity	<i>Stetigkeit</i>
constitution	<i>Beschaffenheit</i>
counteract	<i>entgegenwirken</i>
curvilinear	<i>krummlinicht</i>
decomposition	<i>Scheidung</i>
density	<i>Dichtigkeit</i>
diagonal	<i>Diagonallinie</i>
diffusion	<i>Verbreitung</i>
diminution	<i>Verminderung</i>
direction	<i>Richtung</i>
displacement	<i>Verschiebung</i>
distance	<i>Entfernung</i>
divisibility	<i>Teilbarkeit</i>
division	<i>Teilung</i>
doctrine of appearance	<i>Erscheinungslehre</i>
doctrine of body	<i>Körperlehre</i>
doctrine of motion	<i>Bewegungslehre</i>
doctrine of nature	<i>Naturlehre</i>
effect	<i>Wirkung</i>
elasticity	<i>Elastizität</i>
elastic material	<i>elastische Materie</i>
endure	<i>dauern</i>
enduring presence	<i>daurende Gegenwart</i>

equilibrium	<i>Gleichgewicht</i>
equivalent	<i>gleichgeltend</i>
essence (cf. being)	<i>Wesen</i>
expansive capacity	<i>Ausdehnungsvermögen</i>
expansive force	<i>ausspannende Kraft</i>
experiment	<i>Experiment</i>
extension	<i>Ausdehnung</i>
fermentation	<i>Gärung</i>
figure	<i>Gestalt</i>
fluid	<i>flüssig</i>
force	<i>Kraft</i>
friction	<i>Reibung</i>
fundamental force	<i>Grundkraft</i>
gravitation	<i>Gravitation</i>
gravity	<i>Schwere</i>
ground of determination	<i>Bestimmungsgrund</i>
heat	<i>Wärme</i>
heavenly body	<i>Weltkörper</i>
homogeneous	<i>gleichartig</i>
hydrodynamics	<i>Hydrodynamik</i>
hydrostatics	<i>Hydrostatik</i>
hylozoism	<i>Hylozoismus</i>
hypothesis	<i>Hypothese</i>
impact	<i>Stoss</i>
impart	<i>erteilen</i>
impenetrability	<i>Undurchdringlichkeit</i>
impress	<i>eindrücken</i>
indifferent	<i>gleichgeltend</i>
inertia	<i>Trägheit</i>
interaction	<i>Wechselwirkung</i>
interstices	<i>Zwischenräume</i>
intussusception	<i>Intussuszeption</i>
magnitude	<i>Größe</i>
mass	<i>Masse</i>
material	<i>Stoff</i>
mediate	<i>mittelbar</i>
mode of cognition	<i>Erkenntnisart</i>
mode of explanation	<i>Erklärungsart</i>
mode of representation	<i>Vorstellungsart</i>
monad	<i>Monas</i>
movability	<i>Beweglichkeit</i>

motion	<i>Bewegung</i>
observation	<i>Beobachtung</i>
original	<i>ursprünglich</i>
oscillating	<i>oscillierend, schwankend</i>
particle	<i>Körperchen</i>
perduring	<i>beharrlich</i>
present	<i>darstellen</i>
pressure	<i>Druck</i>
penetrate	<i>durchdringen</i>
penetrate into	<i>eindringen</i>
penetrating force	<i>durchdringende Kraft</i>
penetration	<i>Durchdringung</i>
perception	<i>Wahrnehmung</i>
place	<i>Ort</i>
plurality	<i>Vielheit</i>
porous	<i>löchericht</i>
position	<i>Lage</i>
power	<i>Vermögen</i>
principle	<i>Grundsatz, Princip</i>
progressive	<i>fortschreitend</i>
proportionate	<i>gleichmäßig</i>
quantity	<i>Größe</i>
radius	<i>Halbmesser</i>
rapidity	<i>Schnelligkeit</i>
rational coherence	<i>Vernunftzusammenhang</i>
reaction	<i>Gegenwirkung</i>
reason	<i>Vernunft</i>
reciprocating motion	<i>reziprozierende Bewegung</i>
rectilinear	<i>gradlinicht</i>
representation	<i>Vorstellung</i>
repel	<i>zurücktreiben</i>
repulsion	<i>Zurückstossung</i>
repulsive force	<i>repellierende Kraft, Zurückstossungskraft, zurückstossende Kraft</i>
resistance	<i>Widerstand, Hindernis</i>
rest	<i>Ruhe</i>
rigid	<i>starr</i>
rigidification	<i>Erstarrung</i>
scale	<i>Wagschale</i>
semblance	<i>Schein</i>
separation	<i>Trennung</i>

shape	<i>Gestalt</i>
situation	<i>Lage</i>
speed	<i>Geschwindigkeit</i>
spring	<i>Feder, Springfeder</i>
solid	<i>fest</i>
solidity	<i>Festigkeit, Solidität</i>
sollicitation	<i>Sollicitation</i>
solute	<i>aufzulösende Materie</i>
solvent	<i>Auflösungsmittel</i>
space of activity	<i>Wirkungsraum</i>
sphere of activity	<i>Sphäre der Wirksamkeit</i>
striving	<i>Bestrebung</i>
surface force	<i>Flächenkraft</i>
traction	<i>Zug</i>
transfusion	<i>Transfusion</i>
traverse	<i>zurücklegen</i>
tremor	<i>Bebung</i>
understanding	<i>Verstand</i>
universal attraction	<i>allgemeine Anziehung</i>
vibration	<i>Zitterung, Bebung</i>
vibratory motion	<i>schwankende Bewegung</i>
viscosity	<i>Klebrigkeit</i>
void	<i>die Leere</i>
volume	<i>Volumen, körperlicher Raum</i>
weighing	<i>Abwiegen</i>
weight	<i>Schwere</i>

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